SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION

Approved
No. 0704-0188

AD-A227 366

1a. REPORT SECURITY CLASSIFICATION Unclassified	
2a. SECURITY CLASSIFICATION AUTHORITY	
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	distribution unlimited
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 90 0947
6a. NAME OF PERFORMING ORGANIZATION Washington University 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION AFOSR/NM
6c. ADDRESS (City, State, and ZIP Code) Dept. of Systems Science & Mathematics Box 1040, One Brookings Dr. St. Louis, MO 63130	7b. ADDRESS (City, State, and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448
8a. NAME OF FUNDING / SPONSORING ORGANIZATION AFOSR 8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER Grant # AFOSR 87-0252
8c. ADDRESS (City, State, and ZIP Code) Bldg 410 Bolling AFB DC 20332-6448	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. 61102F PROJECT NO. 2304 TASK NO. A9 WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)

Artificial Intelligence Methods in Pursuit Evasion Differential Games

12. PERSONAL AUTHOR(S)

Rodin, Ervin Y.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final	FROM 6/1/87 TO 5/31/90	1990/7/30	215

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Air Combat; Artificial Intelligence; Differential Games; Semantic Control; Pursuit-Evasion

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This final report contains, as its principal portion, a Technical Report entitled Differential Games and Artificial Intelligence in Air Combat.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. Arje Nachman	22b. TELEPHONE (Include Area Code) (202) 767-4939 22c. OFFICE SYMBOL NM

DD Form 1473, JUN 86 Previous editions are obsolete. SECURITY CLASSIFICATION OF THIS PAGE

AEOSR-TR- 90 0947

FINAL REPORT

Submitted to
Air Force Office of Scientific Research
Building 410, Bolling AFB, DC 20332

by

Ervin Y Rodin, P.I.
Professor and Director

Center for Optimization and Semantic Control
Department of Systems Science and Mathematics
Campus box 1040, Washington University
One Brookings Drive
St. Louis, MO 63130

in connection with

Grant AFOSR-87-0252
Artificial Intelligence Methods in Pursuit Evasion Differential Games

9/3/90

July 30, 1990

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INTRODUCTION

The principal portion of this Final Report on the present grant, **Artificial Intelligence Methods in Pursuit Evasion Differential Games,** consists of an attached Technical Report by Rodin and Weil, on **Differential Games and Artificial Intelligence in Air Combat**. The rest of the report is a brief summary of our activities and achievements under the grant during the past three years. The summary is brief, because it is merely a restatement of reports sent by the P.I. to the AFOSR regularly during the life of the grant. Since those reports also contained copies of publications produced under the sponsorship of this grant, no further copies of such publications are included either. However, we are including copies of a few letters and other material, all of which relate to items of interest mentioned below.

FULFILLMENT OF PROPOSED WORK

In order to gauge whether we succeeded in fulfilling our aims, as described in the Abstract of our proposal to the AFOSR on February 6, 1987, we quote the Abstract:

> *"The aim of the research here proposed is to develop the conceptual framework and the software for the prototype of an operational, on-board, real time multiprocessing computer system, capable of assisting the pilot in flight and fire control decisions; in other words, a Tactical Decision Aiding Expert System (TDAES).*
> *The end product of this research will be for use in theoretical combat planning and analysis; in practical fighter pilot training (e.g., in simulations); and as an actual aid for pilots in support of their tactical decision making during flights.*
> *The nature of this research is intelligent control of a very specific type; we intend to combine certain aspects of differential game theory, 3 dimensional computational geometry and artificial intelligence in a unique way, so as to provide a solution to the problem described above."*

We believe that the attached Technical Report responds in meaningful ways to each of the aims set forth above. Another version of this report also constitutes the doctoral dissertation of R. Weil under the direction of the P.I.; a project which was initiated shortly before the starting date of this grant. Therefore, in an important sense, this report is a culmination and summing up of much of the work we have done during the past three years.The principal subjects of the Report are as follows:

1. Semantic Control Theory
2. Air Combat Environment
3. Pursuit-Evasion Differential Games
4. Artificial Intelligence
5. Numerical Algorithms
6. Multi-objective Decision Making
7. System Integration
8. Simulation

Thus, to be specific, of our aims that we originally set forth, the only one missing is that of multiprocessing. That is a subject we were not able to get into in any meaningful way, because of lack of appropriate computer equipment.

OTHER PROPOSAL FULFILLMENT WORKS

In addition to the Technical Report referred to above, there are other items also, which relate to the fulfillment of our purposes. These include the following:

Doctoral dissertations:

1. Artificial Intelligence Methods in Decision and Control Systems; by Y. Lirov; under the supervision of E.Y. Rodin.

2. Intelligent Prediction Methodologies in the Navigation of Autonomous Vehicles; by M. Amin; under the supervision of E.Y. Rodin.

3. Artificial Intelligence Methods in Utilizing Low Dimensional Models of Differential Games; by R.D. Weil; under the supervision of E.Y. Rodin.

Other publications:

1. "Artificial Intelligence in Air Combat Games" (E.Y. Rodin, L.W. Wilbur, B. McElhany, S. Mittnik and Y. Lirov), **Pursuit-Evasion Differential Games**, ed. by Y. Yavin and M. Pachter, Pergamon Press, pps. 261-274, 1987.

2. "Annotated Bibliography of Pursuit-Evasion Differential Games" (E.Y. Rodin), **Pursuit-Evasion Differential Games**, ed. by Y. Yavin and M. Pachter, Pergamon Press, pps. 275-340, 1987.

3. "Artificial Intelligence Modelling of Control Systems" (E.Y. Rodin, Y. Lirov, B.G. McElhany and L.W. Wilbur), SIMULATION, vol 50, no. 1, pps. 12-24, 1988.

4."Semantic Control Theory" (E.Y. Rodin), Appl. Math. Letters, vol 1, no. 1, pps. 73-78, 1988.

5."Collision Free Path Planning in a Dynamic Environment: Semantic Control Approach" (E.Y. Rodin, B.K. Ghosh, F. Golenko and R.W. Weil), SIMULATION, vol 50, pps.196-201, 1988.

6."Automated Learning by Tactical Decision Systems in Air Combat" (E.Y. Rodin, B.J. Ghosh and Y. Lirov), Comp. and Math. with Appl., vol 18, no.1-3, pps. 151-160, 1989.

7."A Pursuit-Evasion Bibliography - Version 2", (E.Y. Rodin), Comp. and Math. with Appl., vol 18, no. 1-3, pps. 245-320, 1989.

8."Tactical Air Combat Maneuvers: Recognition and Guidance Via Neural Networks" (E.Y. Rodin and M. Amin) , to appear in Aerospace America.

9."Flight and Fire Control with Logic Programming" (E.Y. Rodin and D. Geist), to appear in Comp. and Math. with Appl.

Talks:

In addition to the above paper, there were several talks given by the P.I. and/or his graduate students at various meetings and conferences. Examples of such venues:

Aerospace Applications of Artificial Intelligence Conferences
International Symposia on Differential Games Applications
IEEE Conferences on Decision and Control
IEEE International Symposia on Intelligent Control
Society for Industrial and Applied Mathematics Conferences
Naval Underwater Research Center Seminars
Yale University Seminars
Scott Air Force Base Seminars

BENEFICIAL EFFECTS OF THE GRANT

In addition to the results enumerated above, there were several additional tangible and intangible gains made, which are of potential utility not only to Washington University, but also on a national scale; and gains which, we believe, are of potential use to the USAF . We list some of these below.

Collaboration with Scott AFB:

An excellent working relationship developed, in the course of this grant, between the CINCMAC Analysis Group on the one hand, and the P.I. and his graduate students on the other. Items:

1. A group of 30-40 students of the P.I. now takes a regular annual field trip to Scott AFB, where they are given technical presentations by various AF officers. Several of these students pursue the subjects of these talks in minor and major research projects.
2. We arranged two special seminar series this past school year at Washington University, relating to the applications of Artificial Intelligence, specifically for groups from Scott AFB.
3. Scott AFB officer-scientists gave several special presentations on their problem areas to groups of Washington University students, at the University.
4. A formal agreement of internships was signed between Scott AFB and Washington University.
5. Lt. General A.J. Burshnick nominated the P.I. for membership to the Air Force Scientific Advisory Board.
6. Scott AFB provided funds for the P.I. to equip his laboratory with a SUN 4/260 workstation.

Other Collaboration:

The P.I., his associates and graduate and undergraduate students gave full day progress report type presentations to groups from each of the following organizations:

AFOSR
Scott AFB - CINCMAC Analysis Group
HQ SAC Delegation
Rockwell International
McDonnell Douglas
Emerson Electric

RESEARCH ENVIRONMENT

As a direct result of this grant, the P.I. was enabled to establish at Washington University the following two entities:

Semantic Control Laboratory

This laboratory contains all of the computer equipment, which was utilized for the purposes of this grant, and which will continue to be so utilized. It also contains a large quantity of specialized books and other literature, all of which are specifically earmarked for the type of research here described. The laboratory turned out to be a magnet for many (specifically American undergraduate) students, resulting in increased interest in careers related to science and engineering.

Center for Optimization and Semantic Control

The School of Engineering at Washington University agreed to establish this Center for the specific purpose of both fulfilling the aims of this grant, and for the purpose of enabling the P.I. and his group to continue their research of the nature described here. E.Y. Rodin is Director of the Center; its members are drawn from among the faculty, and from among graduate and undergraduate students.

In addition to the individual and/or joint research activities of members of the Center, an environment is created which is conducive for drawing the interests of some of our brightest, most capable and most motivated students towards our interesting research projects.

The Center holds weekly seminars, at which ongoing research projects, future efforts, etc. are regularly discussed.

ACKNOWLEDGEMENT

The Principal Investigator, Prof. Ervin Y. Rodin, would like to express his thanks to the AFOSR for enabling him and his group of graduate and undergraduate students to engage in the highly rewarding, very interesting and hopefully useful research here described.

DEPARTMENT OF THE AIR FORCE
HEADQUARTERS MILITARY AIRLIFT COMMAND
SCOTT AIR FORCE BASE, ILLINOIS 62225

30 November 1989

Dr James M. McKelvey
Dean, School of Engineering and Applied Science
Washington University
St Louis, Missouri 63130

Dear Dr McKelvey

Just a short note to thank you for your hospitality and the superb presentations of Dr Rodin, his colleagues and students. We were most impressed with the presentations--and the fact that so many students were willing to sacrifice their Thanksgiving holiday to meet with us. They are doing some work of great interest to us in solving real-world problems. We hope to continue this relationship in the future.

Thanks again for taking time out of your busy schedule to meet with us.

Sincerely

JAMES D. GRAHAM, JR., Colonel, USAF
Deputy Director, Command Analysis Group
DCS/Plans and Programs

VOLUNTEER SERVICE AGREEMENT BETWEEN 375 MSSQ/MSCS SCOTT AFB, IL AND WASHINGTON UNIVERSITY, ST. LOUIS, MO

Statement of Understanding. Volunteer service is to be uncompensated and must be required for academic credit or certified to be related to the academic process. Participants are not considered federal employees for any purpose other than for Federal Tort Claims provisions and for purposes relating to compensatory injuries sustained during the performance of work assignments. This service does not lead to noncompetitive permanent employment with this base or with any other federal installation.

Though the university will refer interested students, final selection is left to the Department of the Air Force. We are an equal employment opportunity employer and selection will be based on these principles.

The undersigned parties understand the Volunteer Service Program will provide meaningful experiences reflecting the academic requirements of the participants in a manner so as not to jeopardize other established youth program positions nor positions of any other employee.

<u>Department of the Air Force Volunteer Service Program General Objectives</u>

a. To provide meaningful assignments which relate to the academic endeavors of participants.

b. To provide experiences which enhance the career education and/or career development of participants.

c. To foster amiable relationships between representatives of the Air Force, as a viable potential employer, and university officials.

d. To encourage women, handicapped individuals, and minorities to explore career options available with the Department of the Air Force.

e. To introduce students to the mission and functions of the Air Force and to the work environment so as to facilitate their development of an appreciation for work ethics.

<u>Activity Responsibilities</u>

a. Ensure Office of Personnel Management/Air Force Volunteer Service Program objectives are met.

b. Designate a Volunteer Service Program Coordinator to whom all inquiries can be directed.

c. Provide structured assignments and opportunities for experience which reflects program objectives.

d. Certify and document participation in and completion of the Volunteer Service Program.

e. Provide all participants opportunities to evaluate the program in reference to program objectives.

f. Ensure that this in no way jeopardizes other student employment programs nor contributes to erosion of the position duties of other employees.

Jniversity Responsibilities

ı. Develop a program information dissemination system which ensures that all students are given equal opportunity to apply for referral.

ɔ. Provide activity with name of school officials to whom service inquiries can be directed.

:. Provide students' resumes to activity.

1. Provide information necessary for activity to provide structured, meaningful assignments when academic assignments require such.

e. Inform activity of any change in participants' status.

Participant Responsibilities

a. Though Volunteer Program participants are not federal employees, they are expected to maintain acceptable attendance and to conduct themselves so as not to disrupt normal workflow in the activity.

b. Provide information which may be needed for certain reporting requirements.

Daniel G. Marlett
Civilian Personnel Officer

3/21/90
Date

James M. McKelvey
Dean, School of Engineering
Washington University, St. Louis, MO

3-8-90
Date

THE WASHINGTON UNIVERSITY/MAC INTERN PROGRAM

The Intern Program with Washington University teams an undergraduate student with analysts from the Command Analysis Group on an issue as it is being analyzed for the senior executives at the Military Airlift Command. This teaming provides a symbiotic relationship. The analysts benefit from learning from the student some of the newer innovative methodologies being taught at Washington University. The student experiences the cycle of analysis from the initial nebulous statement of the issue through the presentation of the analytical findings to senior executives for a decision. We want to motivate students to continue a technical course of study by giving them a "piece of the action" and letting them see what practitioners do. We are trying to help alleviate the national problem of low graduate enrollment in the sciences and engineering by American students.

DEPARTMENT OF THE AIR FORCE
HEADQUARTERS STRATEGIC AIR COMMAND
OFFUTT AIR FORCE BASE, NEBRASKA 68113

Ervin Y. Rodin
Box 1040, Washington University
One Brookings Drive
St. Louis, MO 63130-4899

1 9 MAR 1990

Dear Dr Rodin

1. As per your request from our recent telephone conversation, listed below are the individuals planning to visit your facilities.

Dr. Jacqueline Henningsen, Chief, Capability Assessment Division,
Strategic Air Command

Mr. Patrick McKenna, Operations Research Analyst,
Strategic Air Command

Dr. Fred Choobineh, Dept of Industrial Engineering,
University of Nebraska, Lincoln

2. We can arrive at your facilities by 10:00 AM on Monday, April 2, 1990. In line with your suggestion, please send introductory materials describing your program and a possible agenda for the day. We look forward to learning about your progress.

Sincerely

Jackie Henningsen

Dr Jacqueline R. Henningsen, GM-15
Chief, Capability Assessments Div
Dir Force Assessments, DCS/Plans & Resources

Fax # (402) 294-7628
Voice # (402) 294-2355

Washington
WASHINGTON · UNIVERSITY · IN · ST · LOUIS

School of Engineering and Applied Science

Center for Optimization and Semantic Control

To: All SSM (and other interested) undergraduate and graduate students

From: Professor E. Y. Rodin

On Friday, March 2, 1990, a group of scientists from Scott Air Force Base will give a 1-hour talk in the Berger Room (Cupp. II, 101) at 12:00 noon. They will present a set of as yet unsolved optimization problems, which are of great interest to them. Their purpose with the presentation is to interest undergraduate and/or graduate students to undertake joint research with them on some of these problems.

These problems lend themselves to individual (or group) semester or even multi-year projects. They can also be Senior Projects, can be developed to Masters' Theses or could even be the basis for doctoral work.

If you are interested in hearing - without any obligation - what these problems are, please come to the Berger Room at 12:00 noon. You can bring your own lunch. If you cannot make it at 12:00, come at 1:00: if we will have several 1:00 o'clock people, the talks will be repeated for them.

Washington University
Campus Box 1040
St. Louis, Missouri 63130
Tel (314) 889-6007, -5806
FAX (314) 726-4434

CV

Membership Nomination

HQ USAF/NB

1. We are pleased to nominate Dr Ervin Y. Rodin for membership to the Scientific Advisory Board. Dr Rodin is Professor of Applied Mathematics and Systems Science, and Director, Center for Optimization and Semantic Control, Department of Systems Science and Mathematics, Washington University, St Louis MO.

2. Dr Rodin is a leading researcher in scheduling applications, Artificial Intelligence, and Neural Networks. Throughout the past year, Dr Rodin provided exceptional counsel and assistance to the command on airlift operational analysis issues. He is an ideal candidate with his exceptional record and demonstrated willingness to work national issues.

3. Enclosed is Dr Rodin's biographical data. He is aware of his nomination and will gladly serve if selected. Thank you for our opportunity to nominate Dr Rodin.

Sincerely

SIGNED

ANTHONY J. BURSHNICK
Lieutenant General, USAF
Vice Commander in Chief

1 Atch
Dr Rodin's Biographical Data

COORD: XPW

Center for Optimization and Semantic Control

Visit of Program Managers* from
the Air Force Office of Scientific Research
Tuesday, February 13, 1990

PROGRAM

8:30 - 9:00	Ervin Y. Rodin, Director, COSC	Survey of Background, Current Work and Objectives
9:00 - 9:45	Roark Weil, Principal Specialist, McDonnell Douglas Missiles Systems: Graduate Student	Differential Games in Optimal Flight and Fire Control
9:45 - 10:30	Daniel Geist, Graduate Student	Flight and Fire Control with Logic Programming; and Time Varying Voronoi Diagrams
10:30-11:15	Massoud Amin, Graduate Student	Intelligent Prediction Methodologies in the Navigation of Autonomous Vehicles
11:15-12:00	R. Weil, D. Geist and M. Amin	Presentations of Computer Implementations in the Semantic Control Laboratory.
12:00- 1:00	Luncheon Professor Christopher Byrnes, Chairman Department of Systems Science & Math.	Berger Room Education and Research in the Department of Systems Science & Math
1:00 - 1:45	Mark Monical and Chao Ruan, Graduate Students	Further Developments in the Expert Systems Approach to Aeromedical Routing
1:45 - 2:15	A. Girard, D. Horntrop, K. Ruland, Undergraduate Students	Solution Methods for the Operational Support Aircraft Vehicle Scheduling Problem.
2:15 - 3:00	Professor Hiro Mukai and Di Yan, Graduate Student	Optimization with a Probabilistic Performance Index; and Stochastic Approach for Maximum Entropy Distribution
3:00 - 3:30	M. Monical and C. Ruan	Presentation of Computer Implementations in the Semantic Control Laboratory.

*Dr. Charles Holland; Dr. Neil Glassman; Dr. Arje Nachman; Dr. Abe Waxman

We also welcome our other distinguished visitors from HQMAC, Scott AFB; and from the McDonnell Douglas Company.

Center for Optimization and Semantic Control

Visit of Delegation from HQ/SAC-Offut AFB
Monday, April 2, 1990

PROGRAM

10:00-10:15	Ervin Y. Rodin, Director, COSC	Survey of Background, Current Work and Objectives
10:15-11:00	Roark Weil, Principal Specialist, McDonnell Douglas Missiles Systems: Graduate Student	Differential Games in Optimal Flight and Fire Control
11:00-11:45	Massoud Amin, Ph.D.	Intelligent Prediction Methodologies in the Navigation of Autonomous Vehicles
11:45-12:15	R. Weil and M. Amin	Presentations of Computer Implementations in the Semantic Control Laboratory
12:15-1:30	Luncheon	Whittemore House
1:30-2:15	Daniel Geist, Graduate Student	Flight and Fire Control with Logic Programming; and Time Varying Voronoi Diagrams
2:15-3:00	Mark Monical and Chao Ruan, Graduate Students	Further Developments in the Expert Systems Approach to Aeromedical Routing
3:00-3:45	Professor Hiro Mukai and Di Yan, Graduate Student	Optimization with a Probabilistic Performance Index; and Stochastic Approach for Maximum Entropy Distribution
3:45-4:30	D. Geist, M. Monical and C. Ruan	Presentation of Computer Implementations in the Semantic Control Laboratory.

We also welcome our other distinguished visitors from Emerson Electric, Electronics and Space Division; and Rockwell International, North American Aviation.

Center for Optimization and Semantic Control

Prof. Ervin Y. Rodin, Director

Faculty:

M. Amin (Neural Networks, Systems Theory)
B. Ghosh (Robust and Adaptive Systems, Robotics)
R. Loui (Decision Theory and Planning)
H. Mukai (Optimization Methods, Control Systems)
E. Rodin (Artificial Intelligence, Optimization, Differential Games.)

Graduate Students:

D. Geist	R. Weil
J.S. Lin	Y. Wu
M. Monical	D. Yan
C. Ruan	F. Yang

Undergraduates:

D. Baker	J. Hodapp	C. Mayewski
J. Feser	D. Horntrop	A. Rodin
A. Girard	D. Krasnow	M. Ross
C. Hennings	M. Meusey	K. Ruland
		R. Wagnon

Office:

Cupples II, 103
(314) 889-6007

Laboratory:

Cupples II, 111
(314) 889-5806

Fax: (314)726-4434; **e-mail:** rodin@rodin.wustl.edu

Equipment:

SUN 4/260
SUN SparcStation
IBM PC/AT
Macintosh II ci
Macintosh SE 30
1.5 gigabytes of memory

Expansion Plans:

2 SUN SparcStations
Additional Macintosh
Additional Memory
Additional Software

Other Expansion Plans:
Additional faculty, graduate and undergraduate students; and establishment of advisory committee.

Center for Optimization and Semantic Control

Current projects

- Aeromedical Evacuation Problem
- OSA Aircraft Scheduling
- Tactical Decision Aiding Expert System for Flight and Fire Control
- Neural Network Implementation of Decision Making in the Presence of Partial Information
- Neural Network Decision Aiding for Optimal Motion in a Cluttered, Time Varying Environment
- Logic programming

Technical Areas of Interest

- Constrained and unconstrained optimization
- Differential games
- Artificial intelligence
- Neural networks
- Multiprocessing
- Multiobjective decision theory
- Semantic nets
- Multiresolutional algorithms
- Computational complexity
- Delauney triangulation
- Graph theory
- Simulated annealing
- Genetic algorithms

Center for Optimization and Semantic Control

Purpose: To study, in an academic environment, conventional and unconventional optimization and control problems, the genesis of which is in government, industry or business.

Examples:
- Large-scale time varying scheduling problems
- Flight and fire control systems for combat aircraft
- Robotic navigation in cluttered environments

Features:
- Large scale problems (computationally); therefore one of the aims of the analysis must be feasibility and computability.
- Multi-objective optimization problems; sometimes with time varying objectives, sometimes with fuzzy ones.
- Classical optimization, control or game theoretic methods are insufficient for their resolution.

Solution:
- Classical methodologies
- Semantic networks
- Expert Systems
- Neural nets

Additional requirement: User friendliness

Participation:
- Advisory committee
- Multidisciplinary faculty
- Graduate students
- Undergraduate groups

Principal Past Accomplishments:

- Development of Semantic Control paradigms (combination of classical mathematical methodologies with artificial intelligence approaches)

- Development of Tactical Decision Aiding Expert System for Flight and Fire Control (adopted for the Eurofighter)

- Development of Neural Networks Approach to Maneuver Recognition

- Development of Low Observable Tracking Algorithms in Time Varying Environments

TECHNICAL REPORT

Center for Optimization and Semantic Control

WASHINGTON UNIVERSITY

DIFFERENTIAL GAMES AND ARTIFICIAL INTELLIGENCE IN AIR COMBAT*

by

Ervin Y. Rodin

and

Roark David Weil

July, 1990

Saint Louis, Missouri

*Another version of this report was submitted to the Sever Institute of Technology by the second author, under the direction of the first author, in partial fulfillment of his requirements for the degree of Doctor of Science.

ABSTRACT

DIFFERENTIAL GAMES AND ARTIFICIAL INTELLIGENCE IN AIR COMBAT*

by

Ervin Y. Rodin

and

Roark David Weil

A new methodology is developed for dealing with complex systems that are self modifying in the order, relations, and component types. Air-combat is one such system. Differential game technology currently and for the foreseeable future is incapable of generating solutions to all but very low dimensional models for air-combat. The solutions of the low dimensional models are by themselves too simplistic to be useful in actual air-combat.

Artificial intelligence methodologies and the new Semantic Control Paradigm splice together suboptimally the low dimensional differential game solutions. Mesarovic's general systems theory is extended by defining for the first time Semantic Systems dealing with systems that self modify their form (order and relations). A new methodology for simulation of Semantic Systems based on AI Frames is also developed.

Three artificial intelligence methodologies are explored for splicing the differential games: rule based, the Analytical Hierarchy Process, and artificial neural nets. The rule based approach uses an explicit set of rules in a forward or backward chaining fashion to determine the most appropriate differential game. The artificial neural net approach is based on a paradigm modelled after biological processes. Simulation trains a hierarchy of neural nets to determine the most applicable differential game. The Analytical Hierarchy Process is a paradigm of relative measure. First determined is the relative importance of a set of criteria for choosing the most appropriate differential game. Next determined with respect to each criterion is a relative rating of each differential game alternative. A tallied score determines the game.

Those aspects of differential game theory that are useful for air-combat are reviewed. The game of kind determines the region of state space from which

capture of an adversary is possible. The game of degree determines trajectories based on a cost function in the region of state space in which capture is possible. A simplified development of the necessary conditions for the game of degree is given by combining Berkovitz's methodology with Isaac's theorem (that all games are equivalent to an autonomous game with terminal payoff). Berkovitz's result, depended always on using the family of time varying feedback strategies in the variation, has been extended to consider autonomous feedback strategies for autonomous systems.

The overall approach taken in this work has been to combine different technologies that by themselves are currently incapable of solving a problem. Hybridization techniques for future integrated system methodologies may solve problems that are currently beyond the scope of pure solutions. Methodologies in this work have application to problems deemed so difficult that no appropriate mathematical models can be derived, nor can be found any appropriate analytical methods.

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WASHINGTON UNIVERSITY

SEVER INSTITUTE OF TECHNOLOGY

ABSTRACT

ARTIFICIAL INTELLIGENCE METHODS IN UTILIZING LOW DIMENSIONAL MODELS OF DIFFERENTIAL GAMES

by Roark David Weil

ADVISOR: Professor Ervin Y. Rodin

August, 1990

Saint Louis, Missouri

A new methodology is developed for dealing with complex systems that are self modifying in the order, relations, and component types. Air-combat is one such system. Differential game technology currently and for the foreseeable future is incapable of generating solutions to all but very low dimensional models for air-combat. The solutions of the low dimensional models are by themselves too simplistic to be useful in actual air-combat.

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The overall approach taken in this work has been to combine different technologies that by themselves are currently incapable of solving a problem. Hybridization techniques for future integrated system methodologies may solve problems that are currently beyond the scope of pure solutions. Methodologies in this work have application to problems deemed so difficult that no appropriate mathematical models can be derived, nor can be found any appropriate analytical methods.

ARTIFICIAL INTELLIGENCE METHODS IN UTILIZING LOW DIMENSIONAL MODELS OF DIFFERENTIAL GAMES

1. INTRODUCTION

One of the most, if not the most complex dynamical systems is the air-combat arena. The demand on a pilot's faculties becomes more demanding at each incremental increase in the level of technology at both the pilot's and his adversaries' disposal. Thus, the purpose of this research is the development of a real-time tactical flight and fire control system. The development of this system will consider the capabilities of the aircraft, weapons, and electronic countermeasures of both the pilot and his adversaries.

Rufus Isaacs first looked into the theory of modeling tactical encounters in what he termed "Differential Games" in his seminal Rand report. Isaacs assumed a differential model for the aircraft dynamics. He also assumed fixed, the roles of pursuer and evader for the duration of an encounter. Only the pursuer had weapon capabilities. Hyper-surfaces in the state space modeled weapon capabilities. The weapon model supposed that if the state of the system was driven to the aforementioned hyper-surface then the evader is destroyed with probability one, while off the surface there is probability zero that the evader is destroyed.

There are many reasons differential games have not found widespread use in air combat. Isaacs' conditions are only "necessary" conditions. The meeting of these conditions does not guarantee a solution, but a solution must meet these conditions. It has only been possible to find solutions to very low order models for an encounter (e.g., two dimensional constant velocity). This happens because due to the complex geometric structure of the solutions, the controls may be noncontinuous across manifolds in the state space.

The intractability of finding solutions to sufficiently high order models for fixed role encounters is not the only impediment to the use of differential games. The assumption that weapon systems can be modeled by a surface in the state space such that off the surface there is zero probability of destruction of an adversary, and on the surface there is probability one of destruction is not realistic. A continuous probabilistic model that assigns to every point in state space a probability of destroying an adversary is a more realistic weapon systems model. This probability function depends on such items as range, range rate, aspect angle, off boresight angle, etc. Clearly, it is easy to include the effects of electronic countermeasures in this function by adjusting the probabilities in an appropriate fashion. The fixed role assumption during an encounter is also not reasonable. All participants in an encounter have weapon systems that can be modeled by continuous probabilistic functions. Each participant tries to obtain a reasonable firing opportunity while preventing his adversaries from doing the same.

The solution to the low order one target models are by themselves, not sufficiently accurate to be useful in air combat. It is the contention of my research that artificial intelligence methodologies may be used to splice these low order solutions together in a sub-optimal fashion that will be useful in air combat.

The first step in solving the air combat problem is defining how to model such a complex system. The complexity of the air combat arena falls outside current definitions of general systems as first proposed by Mesarovic. The basis for traditional system definitions are a fixed set of objects and a fixed set of relations between the objects. Our interests are systems that have the property that they self modify. This modification takes place by attrition and reproduction of the objects, and modification of relations between the objects. The objects of course are aircraft, missiles, SAMs, and ground targets.

One area of my research explores the characteristics of self modifying systems of the above type. I term these type of systems, semantic systems. Semantic systems are shown to be an extension of general systems defined by Mesarovic. The semantic state object consists of terms representing the order of the system, and the relations among them.

Closed form analytical techniques are generally only useful for very low order system models. Simulation techniques must be used for understanding system models of any real complexity for even classical general systems. The normal case assumes that a system may be classified according to such categories as, continuous dynamical systems, discrete dynamical systems, linguistic systems, decision systems, etc. Closed form analytical tools assume that a system (or its model) is of a homogeneous classification. The problem that we have to deal with is that even the portions of the air-combat arena that can be classified as a Mesarovic general system are not of a homogeneous nature. The fact that the system is self modifying further compounds the problem. I also will introduce a new technique for modeling systems of the aforementioned complex nature. The basis of this technique is Marvin Minsky's artificial intelligence frames. I show that the frame based modeling and simulation paradigm is natural for semantic systems. This allows one to define the systems objects and relations (semantic state) in terms of declarative and procedural knowledge.

The semantic control paradigm will be used to determine the controls for an aircraft. The control problem breaks into three blocks.

(i) Identifier

The identifier fuses sensor data to estimate the current semantic state. Component types (MIGS, SAMS, Airliners) and state are estimated. Also estimated are the relations between the identified components (attacking, retreating). Sensor fusion is itself a complex subject of current research, but is beyond our interest here. We assume complete information of the semantic state.

(ii) Goal Selector

The Goal Selector determines the most appropriate differential game based on the identified semantic state. The choice is made from a knowledge base of games from which solutions can be generated. The results are the optimal trajectories, barrier, and controls. It should be noted that these controls are not the same controls as those needed by the aircraft, because the game chosen may be a simplification of the actual encounter. This block has a library of numerical methods used to solve the differential games.

(iii) Adapter

The Adapter determines the controls that cause the aircraft to "best" follow the optimal trajectory determined by the Goal Selector. This is best accomplished by a rule-based control system dependent upon the differential game model chosen in the identifier, and upon the performance limits of the aircraft.

The design of the goal selector is the main thrust of this dissertation. Three alternate methodologies are considered: rule based, the Analytical Hierarchy Process, and artificial neural nets. The rule based approach uses an explicit set of rules in a forward or backward chaining fashion to find the most appropriate differential game. The artificial neural net approach is based on a paradigm modelled after biological processes. Simulation trains a hierarchy of neural nets to find the most applicable differential game. The Analytical Hierarchy Process is a paradigm of relative measure: First determined is the relative importance of a set of criteria for choosing the most appropriate differential game. Next determined with respect to each criterion is a relative rating of each differential game alternative. A tallied score determines the game.

The use of Pursuit-Evasion games gives us the ability to execute actions in at least locally optimal trajectories, which is something that a human pilot does not automatically achieve. However most Pursuit-Evasion solutions are over simplifications of real life air combat. These solutions need to be refined, and this has to be done when experience is acquired by the utilization of this system.

2. DIFFERENTIAL GAMES

This first section covers the aspects of differential game theory applicable to air-combat. Contained are the derivations of the necessary conditions for the game of kind and game of degree. The derivation of the necessary conditions for the game of degree extends Berkovitz's work to consider only autonomous variations for autonomous systems. Section 6 uses the results derived in this section in generating the solutions for the differential games in the knowledge base.

Rufus Issacs in his seminal Rand Reports first introduced the notion of what he called differential games. He formulated the idea of two opposing players having control of a system described by differential equations. One of the players (pursuer) wishes to drive the state of the system to penetration of a terminal surface (manifold) while the other player (evader) wishes to prevent penetration. Isaacs defined two complementary methodologies of solution to this problem, games of degree and games of kind.

The game of kind determines from what regions of the state space penetration of the terminal surface is possible. The game of kind determines boundaries that separate the state space into regions of capture and escape. The capture region is defined as the set of all points from which the pursuer can force penetration of the terminal surface regardless of the controls used by the evader. Similarly, the escape region is defined as the set of all points from which the evader can prevent penetration of the terminal surface regardless of the controls of the pursuer. The boundaries that separate the capture and escape region are generated by barriers that neither the pursuer nor evader can force to be penetrated. If the barriers do not form the boundary of a closed region of the game space then capture is possible from the entire game space. The barriers then form surfaces that force classical swerve manuevers in the trajectories of the players.

The game of degree determines what the optimal controls and trajectories of the players should be in the capture region. This is done by defining a cost function on the controls, state space, and terminal surface. The solution to the game of degree may not be unique, and exists only for those initial points from which it is possible for the pursuer to drive the state to the terminal surface regardless of the controls of the evader, i.e. for points contained in the capture region determined by the game of kind.

2.1. GAMES OF DEGREE

The necessary conditions for the game of degree are now derived. Our methodology follows Berkovitz (1967) using a theorem of Isaacs (Theorem 2.1.1 page 10) for a simplified presentation. Lemma 2.1.3 and Theorem 2.1.2 are original contributions and extend Berkovitz's work to consider only autonomous variations for autonomous systems. Berkovitz always found it necessary to use the class of time varying variations.

2.1.1. GAME SPACE

Let X be an n-dimensional vector, Y be an s-dimensional vector, and Z be an $s^{\wedge}$-dimensional vector. The game space *E* is a subset of n-dimensional Euclidean space bounded by the terminal manifold *C*. Then $X \in E$ describes the position in the game space and can be described by differential equations:

(2.1.1) $$X_j^{\bullet} = f_j(X,Y,Z); \; j = 1,2,\ldots,n$$

The vector valued function $f(X,Y,Z) = (f_1(X,Y,Z),\ldots,f_n(X,Y,Z))$ is assumed to be of class $C^{(1)}$ on it arguments. The vectors Y and Z are the control vectors of the pursuer and evader respectively.

2.1.2. TERMINAL MANIFOLD

The game ends when the trajectory penetrates an (n-1)-dimensional terminal manifold *C*. The manifold *C* is at least piecewise continuous and on each section is $C^{(1)}$. We assume we can parameterize *C* as:

(2.1.2) $$X = h(s_1,s_2,\ldots,s_{n-1})$$

where h in a n-dimensional vector function.

2.1.3. PAYOFF

The game of degree has integral payoff of the form:

$$P(X_0,\phi,\Phi) = \int_{t_0}^{t_f} G(X,\phi,\Phi)dt + H(s) \tag{2.1.3}$$

where

$H \equiv$ A smooth function defined on (n-1) dimensional terminal manifold C

$G \equiv$ A function containing partials of any order

$\phi \equiv$ The control strategy of the pursuer, a vector valued function $\phi: X \rightarrow Y$

$\Phi \equiv$ The control strategy of the evader, a vector valued function $\Phi: X \rightarrow Z$

$t_f \equiv$ The time the trajectory first contacts the terminal manifold C.

$t_0 \equiv$ Initial time of game.

$X_0 \equiv X(t_0)$

A game of degree is said to have integral payoff if $H \equiv 0$, and terminal payoff if $G \equiv 0$. A game with integral payoff is said to be a pursuit evasion game if $G \equiv 1$, and a game of survival if G is not constant.

The following theorem allows us to derive theoretical results strictly in terms of games of degree with terminal payoff.

<u>THEOREM 2.1.1</u>:[1] A game with payoff $G \neq 0$ can be replaced by an equivalent one with terminal payoff.

This is done by adjoining the new state variable X_{n+1}, new terminal cost H', new terminal manifold $\mathcal{C}'$, and new game space $\mathcal{E}'$ such that:

$$
\begin{aligned}
& X^{\bullet}{}_{n+1} = G(X,\phi,\Phi) \\
& H'(s') = H(s) + s_n \\
& X_{n+1}(t_f) = s_n \text{ (terminal condition)} \\
(2.1.4) \qquad & X_{n+1}(0) = 0 \text{ (initial condition)} \\
& \mathcal{E}' = \mathcal{E} \times (-\infty,\infty) \\
& \mathcal{C}' = \mathcal{C} \times (-\infty,\infty)
\end{aligned}
$$

This theorem can be extended so that our following results are valid for other forms of games not directly incorporated so far. If t effectively appears in f_j, G, or H adjoin $X^{\bullet}{}_{n+1} = 1$, replace t by X_{n+1}, use $X_{n+1}(0) = 0$, $\mathcal{E}' = \mathcal{E} \times (-\infty,\infty)$, and $\mathcal{C}' = \mathcal{C} \times (-\infty,\infty)$. Games of fixed duration can also be incorporated, e.g.

$$(2.1.5) \qquad P(X_0,\phi,\Phi) = \int_0^T G(X,\phi,\Phi)dt$$

This is done by adjoining $T^{\bullet} = X_{n+1} = -1$, $X_{n+1}(0) = T$, $\mathcal{C}' \equiv X_{n+1} = 0$, and $\mathcal{E}' = \mathcal{E} \times (-\infty,\infty)$

The goal of the pursuer will be to minimize the payoff, while the goal of the evader will be to maximize the payoff.

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[1] Isaacs Sec. 2.4, p.g. 1

2.1.4. NECESSARY CONDITIONS

We will now derive the necessary conditions for the optimal strategies of both players. These derivations will follow Berkovitz and we will assume the existence of optimal strategies (feedback control laws). This differs from Isaacs' heuristic treatment in which he only assumed the existence of a single optimal trajectory. Isaacs' theorem that any non-autonomous game with integral cost can be replaced by an equivalent autonoumous game with terminal cost will also be used to simplify the derivations. Berkovitz used non-autonomous games because his variations of strategies required variations in optimality over arbitrarily small increments of time even for autonomous games[2]. We use the notion of level sets to consider only autonomous strategies for autonomous games.

The following assumptions will be needed. Without loss of generality we have shown (Theorem 2.1.1) that only terminal payoff functions need be considered. The payoff function is simply $P(X_0,\phi,\Phi) = H(X_f)$. The closure of E (E^C) is contained in a bounded subset Π of (X)-space. We also assume the existence of the bounded subset Γ of (X,ϕ,Φ)-space which contains the projection of Π. C may be decomposed into C_j ,i = 1, 2,...,α, (n-1)-dimensional connected sub-manifolds of class $C^{(1)}$ each contained in Π such that:

$$C = \bigcup_{j=1}^{\alpha} C_j \tag{2.1.6}$$

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[2]Berkovitz considered the family of non-autonomous strategies even for autonomous games.

The (n-1)-dimensional sub-manifolds are each parameterized as:

(2.1.7) $X = h_j(s_1,s_2,\ldots,s_{n-1})\quad j = 1,2,\ldots,\alpha$

2.1.5. STRATEGIES

The *underlying assumption* is that optimal strategies exist for both the pursuer and evader. Let *U* denote the class of functions ϕ that are piecewise $C^{(1)}$ mappings of X on E^c to s-dimensional Euclidean space. Let *V* denote the class of functions Φ that are piecewise $C^{(1)}$ mappings of X on E^c to $s^\wedge$-dimensional Euclidean space.

Let $\phi \in U$ and $\Phi \in V$, then the differential equations describing the motion in the state space is:

(2.1.8) $X^\bullet = f(X,\phi(X),\Phi(X));\qquad X(t_0) = X_0$

There may be more than one solution to this differential *equation if* X_0 *is point of* discontinuity of ϕ or Φ. A solution that is unique in a neighborhood of X_0 may bifurcate at some future point of the trajectory at a point of discontinuity of ϕ or Φ. Isaacs termed these surfaces of discontinuity of ϕ or Φ semipermeable surfaces. Some of these surfaces to be explored later are barriers, dispersal, and universal surfaces.

Berkovitz defines a playable pair as $\phi \in U$ and $\Phi \in V$ such that for every $X_0 \in E$, X reaches *C* in finite time.[3] This puts the restrictions on the pursuer, as the goal of the evader is to avoid penetration of *C* as long as possible. The underlying assumption is that *E* is the capture region so there must exist at least one ϕ for every Φ that satisfies the playability condition. The payoff function $P(X_0,\phi,\Phi) = H(X_f)$, exists for each playable pair (ϕ, Φ) and $X_n \in E$ but may be

[3]Berkovitz p.g 4

multivalued. The capture region assumption guarantees the non-empty maximal pair of subclasses $U_M \subset U$, $V_M \subset V$ having the property that every pair $\phi \in U_M$, $\Phi \in V_M$ is playable. The members of U_M and V_M are called pure strategies.

The roles in the game are that the pursuer should choose a strategy $\phi \in U_M$ to minimize $P(X_0,\phi,\Phi)$. The evader should choose a strategy $\Phi \in V_M$ to maximize $P(X_0,\phi,\Phi)$. The playable pair (ϕ^*,Φ^*) such that $P(X_0,\phi^*,\Phi^*)$ is single valued is a saddle point if

(2.1.9) $$P(X_0,\phi^*,\Phi) \leq P(X_0,\phi^*,\Phi^*) \leq P(X_0,\phi,\Phi^*)$$

holds for every $X_0 \in E$, $\phi \in U_M$, and $\Phi \in V_M$. The game's optimal strategies are then ϕ^* and Φ^*. The value function of the game is then defined as $W(X_0) = P(X_0,\phi^*,\Phi^*)$. The optimal trajectory corresponding to (ϕ^*,Φ^*) is denoted $\sigma^*(X_0,t)$.

2.'.6. GAME SPACE DECOMPOSITION

We will now look at decomposing the game space into regions of continuity and discontinuity of the optimal controls ϕ^* and Φ^*. The discontinuities are the semi-permeable surfaces mentioned earlier. The semi-permeable surfaces are sub-manifolds of dimension (n-1) and serve to separate the game space E into regions of continuity.

A regular decomposition of the game space E is a decomposition into subregions denoted: E_{11}, E_{12}, ..., E_{1j1}, E_{21}, ..., E_{2j2}, ...,$E_{\alpha 1}$, ..., $E_{\alpha j\alpha}$ such that the E_{ij} satisfy the following properties:[4]

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[4]Remember α is the number of components in the decomposition of the terminal manifold C.

	(a.)	E_{ij}, $i = 1,2,..,\alpha$, $j = 1,2,...,j_i$, is connected and has piecewise smooth boundary.
(2.1.10)	(b.)	$E_{ij} \cap E_{kl} = \emptyset$ $i \neq k$ or $j \neq l$.
	(c.)	$E^C = \bigcup_{i,j} E^C_{ij}$[5]

The regions E_i, $i=1,2,...,\alpha$, defined as

(2.1.11) $$E_i = \bigcup_{j=1}^{j_i} (E^C_{ij} \cap E)$$

are connected, have piecewise smooth boundary, and

(2.1.12) $$E_i \cap E_j = \emptyset \quad i \neq j.$$

The regions E_i $i= 1,2,...,\alpha$ are such that:

(2.1.13)	(a.)	The E_i always lie on the same side of the C_i
	(b.)	$E^C_i \cap C_k = \emptyset$ for $i \neq k$.

We also note that for each $i = 1,2,...,\alpha$:

(2.1.14)	(a.)	$E^C_{i,j_i} \cap C_i \neq \emptyset$
	(b.)	$E^C_{ij} \cap C_i = \emptyset$ for $j \neq j_i$.

The sub-manifolds M_{ij}, $i = 1,2,...,\alpha$, and $j = 1,2,...,j_i-1$ defined by,

(2.1.15) $$M_{ij} = (E^C_{ij} \cap E^C_{i,j+1}) \cap E,$$

are oriented, connected, of dimension (n-1), and of class $C^{(1)}$. We assume the sub-manifolds can be parameterized by $s^j = (s^j_1, s^j_2, ..., s^j_{n-1})$ such that,

(2.1.16) $$X = h_{ij}(s^j_1, s^j_2, ..., s^j_{n-1}).$$

The sub-manifolds M_{ij} then divide E_i into two disjoint regions. The M_{ij}, $i = 1,2,...,\alpha$, and $j,k = 1,2,...,j_i-1$ also satisfy:

(2.1.17) $$M^C_{ij} \cap M^C_{ik} = \emptyset \text{ for } j \neq k.$$

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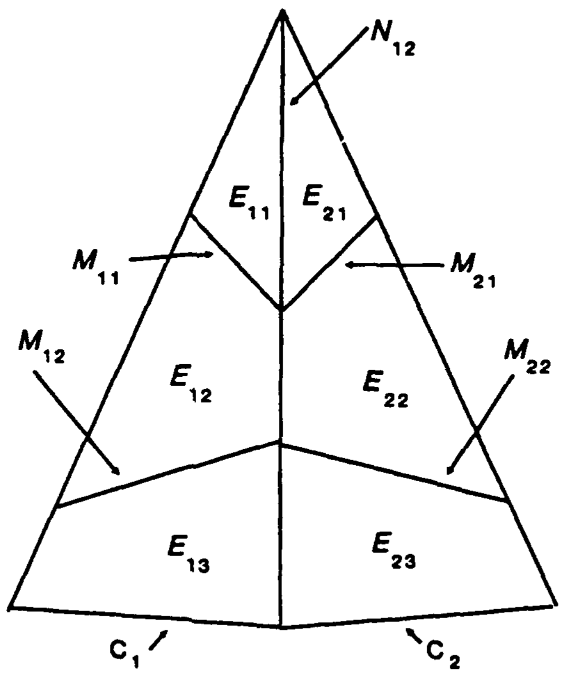
[5]Recall *that* E^C_{ij} is the closure of the set E_{ij}

The sub-manifolds $N_{i1,i2,...,ik}$ defined for each subset $i_1,i_2,...,i_k$ of the integers $i=1,2,...,\alpha$ as

(2.1.18) $N_{i1,i2,...,ik} = (E^c_{i1} \cap E^c_{i2} \cap ... \cap E^c_{ik}) \cap E$ are either empty or a connected non-singular oriented differentiable manifold.



EXAMPLE REGULAR DECOMPOSITION

Some final notation is needed before we close out this section. The sections C_i of the terminal manifold will often be denoted as $M_{i,ji}$. M_{i0} will denote the union of all the $N_{i1,i2,...,ik}$ such that $i \in i_1,i_2,...,i_k$. Lastly for $i = 1,2,...,\alpha$ and $j = 1,2,...,j_i$ we define:

$$E^+_{ij} = E_{ij} \cup M_{ij}$$

(2.1.19) $$E^-_{ij} = E_{ij} \cup M_{i,j-1}$$

$$E^\pm_{ij} = M_{ij} \cup E_{ij} \cup M_{i,j-1}.$$

The following assumptions on the saddle point (ϕ^*,Φ^*) can now be stated:

(2.1.20)

(i) The fact that the functions ϕ^* and Φ^* are piecewise $C^{(1)}$ means there is a regular decomposition associated with the saddle point (ϕ^*,Φ^*) on E with the property ϕ^* and Φ^* are $C^{(1)}$ on E_{ij} for $i = 1,2,\ldots,\alpha$, and $j = 1,2,\ldots,j_i$.

(ii) If $X_0 \in E_{ij}$ then there is a unique optimal trajectory $\sigma^*(X_0,t)$ in E_i for $t_0 < t < t_{i,ji}$ where $t_{i,ji}$ is the time the trajectory reaches C_i. The path is never tangent to sub-manifolds M_{ik}, $k = j,\ldots,j_i$ or $N_{i1,\ldots,ir}$.

Berkovitz[6] points out that if $X_0 \in N_{i1,\ldots,ir}$ then X_0 can be a member of several E_{ij} for several values of i. Then condition (2.1.20) (ii) will hold for each of these values of i. The time derivative of the optimal trajectories $\dot{\sigma}^*$ will in general be *discontinuous at times* t_{ik} *when the trajectory intersects* the sub-manifolds M_{ik}, $k = j,\ldots,j_{i-1}$. This is because either ϕ^* or Φ^* are discontinuous at these points.

2.1.7. VALUE FUNCTION

The following lemma of Berkovitz will be needed before we can discuss the properties of the value of the game. This lemma describes some basic properties of an optimal trajectory, and its intersections with manifolds M_{ik}, $k = j,\ldots,j_{i-1}$.

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[6]Berkovitz pg 6

LEMMA 2.1.1[7] Let $X_0 \in E^{\pm}_{ij}$, $t_{i,k} = t_{i,k}(X_0)$ be the intersect time with manifold M_{ik}, $X_{ik} = X_{ik}(X_0)$ be the intersect point with manifold M_{ik} for $j\text{-}1 \le k \le j_i$. Then for $j \le k \le j_i$, $\sigma^*(X_0,t)$ is defined, is $C^{(1)}$ on $E^{\pm}_{ik} \times [t_{i,k-1},t_{ik}]$, and satisfies:

$$\textbf{(2.1.21)} \qquad X^\bullet = f(X_0,\phi^*(X),\Phi^*(X)); \qquad X(0) = X_0$$

where for $t_{i,k-1}$ and t_{ik} the values are defined as limits from the interior of E_{ik}.

Furthermore for $j \le k \le j_i$ the parameters s^k, t_{ik}, and X_{ik} of manifolds M_{ik} are $C^{(1)}$ functions of X_0 for $X_0 \in E^{\pm}_{ij}$[8].

proof: The optimal controls $\phi^*(X)$ and $\Phi^*(X)$ are by assumption $C^{(1)}$ on E_{ik}, $1 \le k \le k_i$. The fact that E_{ik} is contained in a bounded subset Π implies there exists continuous extensions $\phi^*{}_{ik}(X)$ and $\Phi^*{}_{ik}(X)$ on a region E_{ikE} containing $E^c{}_{ik}$ of the restrictions $\phi^* \mid E_{ik}$ and $\Phi^* \mid E_{ik}$.

We now proceed by induction on each of the regions $E^{\pm}_{ik}$, showing the first assertion is true for $k = j$ where $X_0 \in E^{\pm}_{ij}$. Consider the differential equation for $X_0 \in E^{\pm}_{ij}$ on E_{ijE}:

$$\textbf{(2.1.22)} \qquad X^\bullet = f(X,\phi^*{}_{ij}(X),\Phi^*{}_{ij}(X)); \; X(0) = X_0$$

The existence and uniqueness theorems of ordinary differential equations[9] guarantee that there exists $C^{(1)}$ function $\sigma_{ij}(X_0,t)$ defined for $a(X_0) < t < b(X_0)$ which is a unique solution of the differential equation (2.1.22).

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[7]Berkovitz Lemma 1 pg 8

[8]Recall s^j is the (n-1)-dimensional parameterization of M_{ik} via 2.1.0.

[9]Boothby Theorem 4.1 pg 130

The optimal trajectory is assumed to exist on $E^{\pm}_{ij}$. This along with the fact that (2.1.21) agrees with (2.1.22) on E_{ij} implies that on $E^{\pm}_{ij}$, $\sigma_{ij}(X_0,t) = \sigma^*(X_0,t)$ for $t_{i,j-1} \le t \le t_{ij}$, where t_{ij} is the time the trajectory intersects M_{ij} and $t_{i,j-1}$ is the time the trajectory intersects $M_{i,j-1}$ backwards in time. The first assertion holds for k = j.

We now show the second assertion is true for k = j. The fact that s^j and t_{ij} are $C^{(1)}$ functions of X_0 follows from the implicit function theorem, $\sigma_{ij}(X_0,t_{ij})$ is $C^{(1)}$ on $E_{ijE} \times (t_{i,j-1}-\delta, t_{i,j}+\delta)$[10], h_{ij} is $C^{(1)}$ (2.1.22), the condition:

(2.1.23) $$\sigma_{ij}(X_0,t_{ij}) - h_{ij}(s) = 0$$

and that the nxn matrix

(2.1.24) $$\left[\frac{\partial\sigma^*(X_0,t_{ij})}{\partial t_{ij}} \qquad \frac{-\partial h_{ij}(s^j)}{\partial s^j}\right]$$

has dimension n due to the fact that the optimal trajectory is not tangent to M_j. X_{ij} is a $C^{(1)}$ function of X_0 follows from the fact that h_{ij} is a $C^{(1)}$ function of s^j, s^j is a $C^{(1)}$ function of X_0, and $X_{ij} = h_{ij}(s^j)$. The second assumption holds for k = j.

Assume now that for $j_i \ge k \ge j+1$, $\sigma^*(X_0,t)$ exists and is $C^{(1)}$ on $E^{\pm}_{i,k-1}$. Also that s^{k-1}, $t_{i,k-1}$, $X_{i,k-1}$ are $C^{(1)}$ functions of $X_0 \in E^{\pm}_{ij}$. Let $\alpha \in E^{\pm}_{ik}$, then by similar arguments to those above, there exists $C^{(1)}$ function $\sigma_{ik}(\alpha,t)$ defined for $a(\alpha) < t < b(\alpha)$ which is a unique solution of the differential equation:

(2.1.25) $$X^{\bullet} = f(X,\phi^*_{ik}(X),\Phi^*_{ik}(X)); \; X(0) = \alpha$$

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[10]The fact that the solution $\sigma_{ij}(X_0,t)$ to (2.1.22) exists on $E^{\pm}_{ij} \times [t_{i,j-1},t_{ij}]$ implies it must exist on some open region containing $E^{\pm}_{ij} \times [t_{i,j-1},t_{ij}]$.

Similar arguments to those above imply that for $t_{i,k-1} \le t \le t_{ik}$,

$$\sigma^*(X_{i,k-1}, t - t_{i,k-1}) = \sigma_{ik}(X_{i,k-1}, t - t_{i,k-1}). \tag{2.1.26}$$

On the interval $t_{i,k-1} \le t \le t_{ik}$ we also have by the optimality of the trajectory,

$$\sigma^*(X_0, t) = \sigma^*(X_{i,k-1}, t - t_{i,k-1}) \tag{2.1.27}$$

This leads to the conclusion that for $t_{i,k-1} \le t \le t_{ik}$,

$$\sigma^*(X_0, t) = \sigma_{ik}(X_{i,k-1}, t - t_{i,k-1}) \tag{2.1.28}$$

We have shown that σ_{ik} is $C^{(1)}$ on $\mathcal{E}^{\pm}_{ik} \times [t_{i,k-1}, t_{ik}]$, and by assumption $X_{i,k-1}$ and $t_{i,k-1}$ are $C^{(1)}$ functions of X_0. This implies that $\sigma^*(X_0,t)$ is $C^{(1)}$ on $\mathcal{E}^{\pm}_{ij} \times [t_{i,k-1}, t_{ik}]$.

Last we must show that s^{k-1}, $t_{i,k-1}$, $X_{i,k-1}$ are $C^{(1)}$ functions of $X_0 \in \mathcal{E}^{\pm}_{ij}$. Since σ_{ik} is $C^{(1)}$ on $\mathcal{E}_{ikE} \times (t_{i,k-1}-\delta, t_{ik}+\delta)$ we can repeat arguments for with (2.1.23) and (2.1.24) where we simply replace j by k.

We now consider the properties of the value function $W(X_0) = P(X_0, \phi^*, \Phi^*)$. The following lemma of Berkovitz states the smoothness properties of $W(X_0)$. We note that W is continuous on $\mathcal{E}_{ij}$.

<u>**LEMMA 2.1.2**</u>:[11] $W(X_0)$ is a $C^{(1)}$ function of $X_0 \in \mathcal{E}^{\pm}_{ij}$, and is continuous on $\mathcal{E}$.

proof: The value function can be written in terms of the contact of the terminal manifold $\mathcal{C}$:

$$W(X_0) = P(X_0, \phi^*, \Phi^*) = H(X_{i,ji}). \tag{2.1.29}$$

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[11] Berkovitz pg 16

The fact that H is $C^{(1)}$ by assumption, and $X_{i,ji}$ is a $C^{(1)}$ function of $X_0 \in \mathcal{E}^{\pm}_{ij}$ (Lemma 2.1.1) implies the $C^{(1)}$ assertion.

Since W is continuous on $\mathcal{E}^{\pm}_{ij}$ we need only show for continuity on $\mathcal{E}_i$ that for every $X \in \mathcal{M}_{ij}$ there exists a sequence from each side of $\mathcal{M}_{ij}$ approaching X whose limits are equal.[12] Let $X_0 \in \mathcal{E}_{ij}$ such that $\sigma^*(X_0,t_{ij}) = X$, by Lemma 2.1.1 such an X_0 must exist. Along any trajectory $W(\sigma^*(X_0,t))$ is constant implying $W(\sigma^*(X_0,t^-_{ij})) = W(\sigma^*(X_0,t^+_{ij}))$.

The assumption that the payoff function is single valued for the optimal trajectories is used to extend the continuity of W(X) to all of the game space $\mathcal{E}$. Let $X \in \mathcal{N}_{1,2,\ldots,k}$, and $\sigma^*_1(X,t),\ldots,\sigma^*_K(X,t)$ be the corresponding optimal trajectories in each region $\mathcal{E}_1,\ldots,\mathcal{E}_K$. Since $\sigma^*_1(X,0^+) = \sigma^*_2(X,0^+) = \ldots = \sigma^*_K(X,0^+)$, and W(X) is continous on all $\mathcal{E}_i$ the conclusion holds.

The following notation needs to be introduced. If $X_0 \in \mathcal{M}_{ij}$, then $W^-_X(X_0) = \lim W_X(X)$ as $X \to X_0$ from $\mathcal{E}_{ij}$. Likewise, $W^+_X(X_0) = \lim W_X(X)$ as $X \to X_0$ from $\mathcal{E}_{i,j+1}$. Lemma 2.1.2 guarantees the existence of these one sided limits.

One more lemma is needed before the main result of this section can be handled. This lemma states the properties of variations from the optimal controls. The lemma is new and allows the extension of Berkovitz's work to consider only the family of autonomous pure strategies for autonomous systems.

<u>LEMMA 2.1.3</u>: Let $\phi \in \mathcal{U}_M$, $X_0 \in \mathcal{E}_{ij}$ and $R > 0$. Denoting the trajectory corresponding to ϕ,Φ^* by $\sigma(t,X)$ we can define the set:

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[12]Since we have already shown continuity on $\mathcal{E}^{\pm}_{ij}$ we do not have to show this for every sequence.

$$(2.1.30)\qquad N_R(X_0) = \{\, X : W(X) \le W(\sigma(R,X_0)) \,\}.$$

Then $N_R(X_0)$ satisfies the following properties,

$$\begin{array}{lll} & \text{i.)} & N_R(X_0) \text{ is closed,} \\ (2.1.31) & \text{ii.)} & X_0 \in N_R(X_0), \\ & \text{iii.)} & \text{If } X \notin N_R(X_0) \text{ then } \sigma^*(t,X) \notin N_R(X_0). \end{array}$$

Define for $\phi \in U_M$ a corresponding ϕ_R by:

$$(2.1.32)\qquad \phi_R(X) = \begin{cases} \phi^*(X) & \text{if } X \notin N_R(X_0) \\ \phi(X) & \text{if } X \in N_R(X_0). \end{cases}$$

Then iv.) $\phi_R \in U_M$. The trajectory corresponding to ϕ_R, Φ^* is denoted by $\sigma_R(t,X)$.

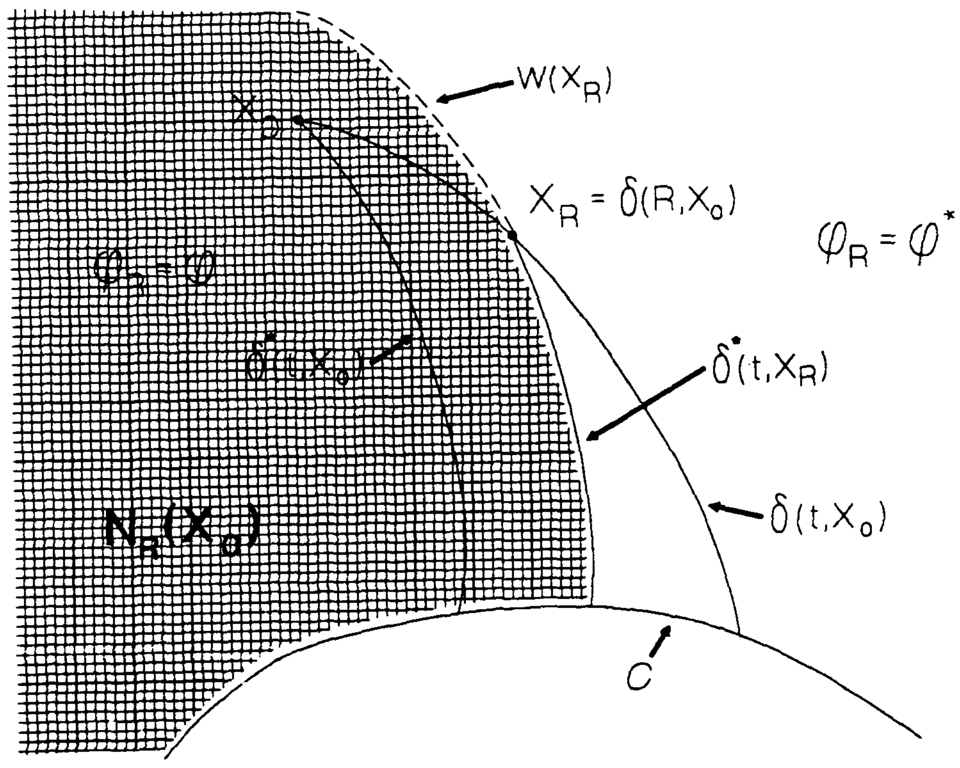

ILLUSTRATION OF $N_R(X_0)$ & φ_R

proof: The properties i.) and ii.) follow easily from the continuity of $W(X)$. Property iii.) follows from the optimality of the strategies. Assume there exists an $X \in N_R(X_0)^{comp}$ and $t > 0$ such that $\sigma^*(t,X) \in N_R(X_0)$.[13] The fact the we are only dealing with terminal costs implies that $W(X) = W(\sigma^*(t,X))$ hence $X \in N_R(X_0)$: a contradiction.

We now turn our attention to property iv.). There are two cases to be considered. The first case is when the optimal trajectory never leaves $N_R(X_0)$. Then $\sigma_R(t,X) = \sigma(t,X)$ for $t \geq 0$, implying the trajectory intersects the terminal manifold in finite time due to the admissibility of $\phi(X)$. If the trajectory leaves $N_R(X_0)$ at time T then $\sigma_R(t,X) = \sigma^*(t-T,\sigma(T,X))$ for $t \geq T$, implying the trajectory intersects the terminal manifold in finite time.

Isaacs' equation can now be derived in the following theorem. This theorem is *new and considers only variations of autonomous strategies.*

<u>**THEOREM** 2.1.2</u>: On $\mathcal{E}^{\pm}_{ij}$ the following equation is satisfied for all $\phi \in \mathcal{U}_M$ and $\Phi \in \mathcal{V}_M$:[14]

$$\underset{\phi}{\mathrm{Min}}\,\underset{\Phi}{\mathrm{Max}}(W_X(X)f(X,\phi,\Phi)) = \underset{\Phi}{\mathrm{Max}}\,\underset{\phi}{\mathrm{Min}}(W_X(X)f(X,\phi,\Phi)) = 0$$

where for $X \in \mathcal{M}_{ij}, \mathcal{M}_{i,j-1}$ the equation is defined as a limit from the interior of $\mathcal{E}_{ij}$.

proof: Let $X_0 \in \mathcal{E}_{ij}$, and $\phi \in \mathcal{U}_M$ then by Lemma 2.1.3, $N_R(X_0)$ and ϕ_R are properly defined for $R > 0$. The assumption that ϕ^*,Φ^* is a saddle point implies that:

(2.1.33) $$W(X_0) = P(X_0,\phi^*,\Phi^*) \leq P(X_0,\phi_R,\Phi^*) = W(\sigma_R(R,X_0))$$

[13] $N_R(X_0)^{comp}$ is the complement of $N_R(X_0)$.

[14] W_X is the partial derivative vector of W wrt. X.

where σ_R is the trajectory corresponding to ϕ_R, Φ^* and R is the exit time from $N_R(X_0)$.

The condition (2.1.33) implies that $W(X_0) - W(\sigma_R(R,X_0)) \leq 0$ with equality for $\phi = \phi^*$. Lemma 2.1.2 and Lemma 2.1.1 state that W and σ_R are both $C^{(1)}$ so the mean value theorem may be applied twice on $N_R(X_0)$ using the fact that $X_0 = \sigma_R(0,X_0)$.

(2.1.34)
$$\begin{aligned} & W(X_0) - W(\sigma_R(R,X_0)) \\ &= -W_X(X_0)(\sigma_R(R,X_0) - X_0) + O(\sigma_R(R,X_0)-X_0)^2 \\ &= -W_X(X_0)(\sigma^\bullet{}_R(0,X_0)R + O(R)^2) + O(R)^2 \\ &= -W_X(X_0)f(X_0,\phi(X_0),\Phi^*(X_0))R + O(R)^2 \\ &\leq 0 \ \text{\{by equation (2.1.33)\}} \end{aligned}$$

Dividing equation (2.1.34) thru by R, we have in the limit as $R \to 0$:

(2.1.35) $\quad W_X(X_0)f(X_0,\phi(X_0),\Phi^*(X_0)) \geq 0$ for every $\phi \in \mathcal{U}_M$

with equality for $\phi = \phi^*$.

Similar arguments can show for Φ that:

(2.1.36) $\quad W_X(X_0)f(X_0,\phi^*(X_0),\Phi(X_0)) \leq 0$ for every $\Phi \in \mathcal{V}_M$

with equality for $\Phi = \Phi^*$.

Combining equation (2.1.35) and (2.1.36) we have:

(2.1.37)
$$\begin{aligned} & W_X(X_0)f(X_0,\phi^*(X_0),\Phi(X_0)) \\ & \qquad \leq W_X(X_0)f(X_0,\phi^*(X_0),\Phi^*(X_0)) \\ & \qquad \leq W_X(X_0)f(X_0,\phi(X_0),\Phi^*(X_0)) \end{aligned}$$

for every $\phi \in \mathcal{U}_M$, $\Phi \in \mathcal{V}_M$. Equality holds for $\phi = \phi^*$ and $\Phi = \Phi^*$ simultaneously.

Equation (2.1.37) implies the assertion of the theorem, (2.1.32) on $\mathcal{E}_{ij}$. We now extend this result to $\mathcal{E}^{\pm}_{ij}$. Without loss of generality assume $X \in \mathcal{M}_{ij}$, then due to Lemma 2.1.2 and the fact that f is $C^{(1)}$ by assumption, the limit of equation (2.1.37) as $X_0 \to X$ from E_{ij} exists and implies:

(2.1.38) $$W^-_X(X)f(X,\phi^*_{ij}(X),\Phi(X)) \le W^-_X(X)f(X,\phi^*_{ij}(X),\Phi^*_{ij}(X)) \le W^-_X(X)f(X,\phi(X),\Phi^*_{ij}(X)),$$

where the continuous extensions of the controls defined earlier are used.

The last question to be tackled in this section is the continuity of W_X across manifolds M_{ij}. The following theorem of Berkovitz handles this question in the case both ϕ^* and Φ^* are not discontinuous at M_{ij}.

THEOREM 2.1.3:[15] If ϕ^* and Φ^* are both not discontinuous at M_{ij} then W_X is continuous across M_{ij}.

proof: *The assertion will be proven if we can show* $W^-_X(X_0) = W^+_X(X_0)$. The fact that W is $C^{(1)}$ on $E^{\pm}_{ij}$ (Lemma 2.1.2), and M_{ij} is a $C^{(1)}$ manifold implies (similarly to Lemma 2.1.1) that there exists a $C^{(1)}$ extensions $W_{ij}(X)$ on a region E_{ijE} containing E^c_{ij} of the restriction $W \mid E_{ij}$. Given arbitrary $X_0 \in M_{ij}$ and letting x(r) be any $C^{(1)}$ curve lying in M_{ij} such that x(0) = X_0 we have $w(r) = W(x(r)) = W_{ij}(x(r))$. Then the derivative of w(r) at r = 0 is:

(2.1.39) $$w^\bullet(0) = W_{Xij}(X_0)x^\bullet(0) = W^-_X(X_0)x^\bullet(0).$$

Similar arguments can show that,

(2.1.40) $$w^\bullet(0) = W^+_X(X_0)x^\bullet(0).$$

This leads to

(2.1.41) $$(W^-_X(X_0) - W^+_X(X_0))x^\bullet(0) = 0.$$

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[15]Berkovitz Theorem 1 pg 16

The fact that $x(r)$ is an arbitrary curve and $\mathcal{M}_{ij}$ is a (n-1)-dimensional manifold implies $W^+{}_X(X_0) = W^-{}_X(X_0)$ or $(W^-{}_X(X_0) - W^+{}_X(X_0))$ is nonzero and orthogonal to $\mathcal{M}_{ij}$ at X_0. Assume the latter and without loss of generality that $\Phi^*(X)$ is continuous across then $\mathcal{M}_{ij}$. Analogous to Lemma 2.1.3 we define the following function on $N_R(X_0)$:

$$\text{(2.1.42)} \qquad \phi_{ijR}(X) = \begin{cases} \phi^*(X) \text{ if } X \notin N_R(X_0) \\ \phi^*{}_{ij}(X) \text{ if } X \in N_R(X_0), \end{cases}$$

is a member of $\mathcal{U}_M$, where $\phi^*{}_{ij}(X)$ is again the continuous extension of the control as defined earlier.

The results of Theorem 2.1.1 on $\mathcal{E}^{\pm}{}_{ij}$ (2.1.32) imply that:

$$\text{(2.1.43)} \qquad \begin{aligned} 0 &= W^-{}_X(X_0)f(X_0,\phi^*{}_{ij}(X_0),\Phi^*(X_0)) \\ &\le W^-{}_X(X_0)f(X_0,\phi_{i,j+1,R}(X_0),\Phi^*(X_0)), \\ 0 &= W^+{}_X(X_0)f(X_0,\phi^*{}_{i,j+1}(X_0),\Phi^*(X_0)) \\ &\le W^+{}_X(X_0)f(X_0,\phi_{ijR}(X_0),\Phi^*(X_0)). \end{aligned}$$

Using the fact that $\phi_{ijR}(X_0) = \phi^*{}_{ij}(X_0)$ and combining equations of (2.1.42) we get:

$$\text{(2.1.44)} \qquad \begin{aligned} &(W^-{}_X(X_0) - W^+{}_X(X_0))f(X_0,\phi^*{}_{ij}(X_0),\Phi^*(X_0)) \le 0, \\ &(W^-{}_X(X_0) - W^+{}_X(X_0))f(X_0,\phi^*{}_{i,j+1}(X_0),\Phi^*(X_0)) \ge 0. \end{aligned}$$

The inequality must hold in equations (2.1.43), otherwise due to the assumption that $(W^-{}_X(X_0) - W^+{}_X(X_0))$ is nonzero and orthogonal to $\mathcal{M}_{ij}$, $f(X_0,\phi_{ij}(X_0),\Phi^*(X_0))$ and $f(X_0,\phi_{i,j+1}(X_0),\Phi^*(X_0))$ must be tangent to $\mathcal{M}_{ij}$. If the inequality holds then on each side of the tangent plane of $\mathcal{M}_{ij}$ at X_0 the trajectory points into opposite sides of the half space, an impossibility. This implies $W^-{}_X(X_0) = W^+{}_X(X_0)$.

2.1.8. ADJOINT EQUATIONS

We will now derive the adjoint equations following Berkovitz. The fact that the ajoint equations are derived in retro-time from the terminal manifold, and the optimal trajectories are defined only in forward time for $t \geq 0$ causes us to first introduce and define the function $\Gamma(t,X_0)$. Then we show for $t \geq 0$:

(2.1.45)
$$\frac{\partial W(X)}{\partial X} = W_X(X) = \Gamma(t,X_0),$$

with $X = \sigma^*(t,X_0)$.

<u>LEMMA **2.1.4**</u>:[16] Let $X_0 \in \mathcal{E}^{\pm}_{ij}$, define the function $\Gamma(t,X_0)$ as follows. On each interval $[t_{i,k-1}, t_{ik}]$, $j \leq k \leq j_i$, Γ satisfies the following differential equation:

(2.1.46)

$$\Gamma^{\bullet} = -\Gamma \left[\frac{\partial f(\sigma^*(t,X_0),\phi^*(\sigma^*(t,X_0)),\Phi^*(\sigma^*(t,X_0)))}{\partial X} \right.$$

$$+ \frac{\partial f(\sigma^*(t,X_0),\phi^*(\sigma^*(t,X_0)),\Phi^*(\sigma^*(t,X_0)))}{\partial \phi} \frac{\partial \phi^*(\sigma^*(t,X_0))}{\partial X}$$

$$\left. + \frac{\partial f(\sigma^*(t,X_0),\phi^*(\sigma^*(t,X_0)),\Phi^*(\sigma^*(t,X0)))}{\partial \Phi} \frac{\partial \Phi^*(\sigma^*(t,X_0))}{\partial X} \right]$$

with the initial condition $\Gamma(t^-_{iK},X_{iK})$, and final condition $\Gamma(t^+_{i,K-1},X_{i,K-1})$. The initial conditions on each interval are described as follows.

[16]Berkovitz Lemma 4 pg 19

Define $\Gamma(t^-_{iK}) = \Gamma^-_{iK}$ where for $k = j_i$, $\Gamma^-_{i,ji} = \Gamma_{ji}$, where Γ_{ji} is defined by the n linear equations[17]:

$$\text{(2.1.47)} \qquad \Gamma_{ji} \begin{bmatrix} \dfrac{\partial X_{i,ji}}{\partial s} & -\sigma^{*\bullet}(t^-_{i,ji}, X_0) \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\partial H(X_{i,ji})}{\partial X} \dfrac{\partial X_{i,ji}}{\partial s} & 0 \end{bmatrix}.$$

If $k < j_i$ then letting $\Gamma^+_{iK} = \Gamma(t^+_{iK})$, Γ^-_{iK} is defined by the following n linear equations:

$$\text{(2.1.48)} \qquad \Gamma^-_{ik} \begin{bmatrix} \dfrac{\partial X_{ik}}{\partial s} & -\sigma^{*\bullet}(t^-_{iK}, X_0) \end{bmatrix}$$

$$= \Gamma^+_{ik} \begin{bmatrix} \dfrac{\partial X_{ik}}{\partial s} & -\sigma^{*\bullet}(t^+_{iK}, X_0) \end{bmatrix}.$$

Then on each interval $[t_{i,k-1}, t_{ik}]$, Γ is well defined and a continuous function of t and X_0 for $0 \le t$ and $j \le k \le j_i$.

proof: The proof will be by induction. We first show the lemma holds for k $= j_i$. The matrix on the left hand side of equation (2.1.47) has dimension n due to the non-tangency condition. This implies Γ_{ji} is uniquely defined. The fact that

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[17] These are known as the transversality conditions.

$X_{i,ji}$ is a $C^{(1)}$ function of s, and Lemma 2.1.1 imply that (2.1.47) uniquely defines Γ_{ji} as a continuous function of X_0.

Equation (2.1.46) defines an n-dimensional linear system in Γ. The fact that the matrix on the RHS of equation (2.1.46) is a continuous function of t for $t_{i,ji-1} \le t \le t_{i,ji}$ implies the unique solution $\Gamma(t,X_0)$ exists on this interval[18]. The conclusion then holds for $k = j_i$.

We now show that if the conclusion holds for k + 1, it holds for k. Equation (2.1.48) uniquely defines Γ^-_{iK} since the non-tangency condition implies the matrices of (2.1.48) are non-singular. The induction assumption implying Γ^+_{iK} is a continuous function of X_0, along with Lemma 2.1.1 imply that Γ^-_K is a continuous function of X_0. Then differential equation (2.1.46) is well defined similar to $k = j_i$ with the new initial condition $\Gamma(t^-_K) = \Gamma^-_K$ for $t_{i,K-1} \le t \le t_{iK}$. The *unique solution* $\Gamma(t,X_0)$ exists on this interval.

<u>THEOREM 2.1.4:</u>[19] Let $X_0 \in E_{ij}$, $\Gamma(t,X_0)$ be the function described in Lemma 2.1.4, if $X = \sigma^*(t,X_0)$ then for $t \ge 0$:

(2.1.49) $\quad W_X(X) = \Gamma(t,X_0).$

proof: Let $X_0 \in E_{ij}$, because $W(X_0) = H(X_{i,ji})$, and from Lemma 2.1.1 parameter s^j is a $C^{(1)}$ function of X_0 we have

(2.1.50) $$W_{X0}(X_0) = \frac{\partial H(X_{i,ji})}{\partial X_{i,ji}} \frac{\partial X_{i,ji}}{\partial s} \frac{\partial s}{\partial X_0}$$

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[18]Zadeh and Desoer, 1963

[19]Berkovitz Theorem 2 pg 21

Define F as the n x n matrix in equation (2.1.46), so that we can write $\Gamma^{\bullet} = -\Gamma F$.

Then we have

(2.1.51) $$W_{X0}(X_0) = \frac{\partial H(X_{i,ji})}{\partial X_{i,ji}} \frac{\partial X_{i,ji}}{\partial s} \frac{\partial s}{\partial X_0}$$

$$- \sum_{k=j}^{j_i-1} \int_{t_{i,K}}^{t_{i,K+1}} (\Gamma^{\bullet} + \Gamma F) \frac{\partial \sigma^*}{\partial X_0} dt$$

$$- \int_0^{t_{ij}} (\Gamma^{\bullet} + \Gamma F) \frac{\partial \sigma^*}{\partial X_0} dt$$

$$= \frac{\partial H(X_{i,ji})}{\partial X_{i,ji}} \frac{\partial X_{i,ji}}{\partial s} \frac{\partial s}{\partial X_0}$$

$$- \sum_{k=j}^{j_i-1} \int_{t_{i,K}}^{t_{i,K+1}} d\left[\Gamma \frac{\partial \sigma^*}{\partial X_0}\right] - \int_0^{t_{ij}} d\left[\Gamma \frac{\partial \sigma^*}{\partial X_0}\right]$$

$$= \frac{\partial H(X_{i,ji})}{\partial X_{i,ji}} \frac{\partial X_{i,ji}}{\partial s} \frac{\partial s}{\partial X_0}$$

$$+ \Gamma(0,X_0) \frac{\partial \sigma^*(0,X_0)}{\partial X_0} - \Gamma_{ji} \frac{\partial \sigma^*(t^-_{i,ji},X_0)}{\partial X_0} +$$

$$\sum_{k=j}^{j_i-1} \left[\Gamma^+_{iK} \frac{\partial \sigma^*(t^+_{iK},X_0)}{\partial X_0} - \Gamma_{ik} \frac{\partial \sigma^*(t^-_{iK},X_0)}{\partial X_0}\right]$$

Let us now look at the terms in the summation above. Expanding the partials of the optimal trajectory, and using equation (2.1.48), we have:

(2.1.52)
$$\sum_{k=j}^{j_i-1}\left[\Gamma^+_{iK}\frac{\partial\sigma^*(t^+_{iK},X_0)}{\partial X_0} - \Gamma^-_{ik}\frac{\partial\sigma^*(t^-_{iK},X_0)}{\partial X_0}\right]$$

$$=\sum_{k=j}^{j_i-1}\left[\left[\Gamma^+_{iK}\sigma^{*\bullet}(t^+_{iK},X_0) - \Gamma^-_{ik}\sigma^{*\bullet}(t^-_{iK},X_0)\right]\frac{\partial t_{ik}}{\partial X_0} + \left[\Gamma^+_{iK}\frac{\partial X_{i,k}}{\partial s} - \Gamma^-_{ik}\frac{\partial X_{i,k}}{\partial s}\right]\frac{\partial s}{\partial X_0}\right]$$

$$= 0.$$

We can also use equation (2.1.47) to show

(2.1.53)
$$\Gamma_{ji}\;\frac{\partial\sigma^*(t^-_{i,ji},X_0)}{\partial X_0}$$

$$= \Gamma_{ji}\left[\sigma^{*\bullet}(t^+_{i,ji},X_0)\frac{\partial t_{i,ji}}{\partial X_0} + \frac{\partial X_{i,ji}}{\partial s}\frac{\partial s}{\partial X_0}\right]$$

$$= \frac{\partial H(X_{i,ji})}{\partial X_{i,ji}}\;\frac{\partial X_{i,ji}}{\partial s}\;\frac{\partial s}{\partial X_0}.$$

Combining equations (2.1.51), (2.1.52), and (2.1.53) along with the known fact that

(2.1.54)
$$\frac{\partial\sigma^*(0,X_0)}{\partial X_0} = I \text{ (nxn identity matrix)},$$

we have $W_{X0}(X_0) = \Gamma(0,X_0)$.

We note that if $X = \sigma^*(\tau, X_0)$ then $\sigma^*(t, X_0) = \sigma^*(t-\tau, X)$ for $t \geq \tau$. This implies $\Gamma(t, X_0) = \Gamma(t-\tau, X)$ for $t \geq \tau$ due to equation (2.1.46). Hence $W_X(X) = \Gamma(0, X) = \Gamma(t, X_0)$.

2.2. GAMES OF KIND

The achievement of termination (reaching terminal manifold *C*) is the kernel of the problem. We wish to determine from what positions in the game space *E* capture can be guaranteed. The game space *E* is partitioned into the capture region and escape region. The surface that separates these two regions is termed the barrier *(B)*. This section will be concerned with the derivation of the equations that generate the barrier, and follows Isaacs' heuristic treatment found in his Rand report.

2.2.1. FEASIBLE CONTROLS

At each $X \in E$ we assume defined a set of feasible controls for the pursuer $U(X)$ and evader $V(X)$. The pursuer is free to pick any control vector $\phi \in U(X)$. The evader is free to pick any control vector $\Phi \in V(X)$.

2.2.2. TERMINAL MANIFOLD

The terminal manifold (*C*) is an (n-1)-dimensional manifold. We assume the manifold can be parameterized by s_i; i=1,.., n-1 and the coordinate functions h_j; j = 1,...,n such that,

(2.2.1) $$X_j = h_j(s_1,..,s_{n-1});\ j = 1,..,n.$$

The condition of capture is defined only when the terminal manifold (*C*) is penetrated. The condition when a trajectory X reaches *C* without penetration and returns to *E* will be considered neutral. This neutral condition delineates capture and escape. Defining $\underline{\nu} = (\underline{\nu}_1,...,\underline{\nu}_n)$ to be the normal to *C* at x extending into *E*, the condition,

(2.2.2) $$\min_{\phi} \max_{\Phi} \sum_{i=1}^{n} \underline{\nu}_i(x) f_i(x,\phi,\Phi) > 0,$$

implies that the evader E can prevent immediate termination from a position sufficiently close to *C*. The region of *C* where the above holds is referred to as the non-usable part (NUP) of *C*. The opposite condition

(2.2.3) $$\min_{\phi}\max_{\Phi}\sum_{i=1}^{n} \underline{\nu}_i(x)f_i(x,\phi,\Phi) < 0$$

implies that the pursuer P can force termination for positions sufficiently close to *C*. The region of *C* where this condition holds is referred to as the usable part (UP) of *C*. The neutral condition can only hold for trajectories containing a member x of *C* such that the condition,

(2.2.4) $$\min_{\phi}\max_{\Phi}\sum_{i=1}^{n} \underline{\nu}_i(x)f_i(x,\phi,\Phi) = 0.$$

The region of *C* where this condition holds is referred to as the boundary of the usable part of *C* (BUP). *The condition* for the BUP is given on an (n-1)-dimensional manifold, hence the BUP is an (n-2)-dimensional manifold. We seek to attach to the BUP a surface that neither the pursuer or evader can penetrate and that separates *E* into the capture and escape region. We begin by discussing these semipermeable surfaces of dimension (n-1) and how they may be attached to (n-2) dimensional manifolds.

2.2.3 SEMIPERMEABLE SURFACES

Let *S* be an orientable surface separating the neighboring space. The two directions of penetration of *S* are termed the P-direction and E-direction. The "side" of the surface *S* reached after penetration in the P-direction is termed the P-side. The definition of the E-side follows naturally.

Definition: A surface S is termed semipermeable at X when,

i.) There exists a value $\phi^* \in U(X)$ (the controls of P) such that when $Y = \phi^*$, no vector $\Phi \in V(X)$ causes $f(X,\phi^*,\Phi)$ to penetrate the surface S in the E-direction,

ii.) There exists a value $\Phi^* \in V(X)$ (the controls of E) such that when $Z = \Phi^*$, no vector $\phi \in U(X)$ causes $f(X,\phi,\Phi^*)$ to penetrate the surface S in the P-direction.

A surface S is termed a semipermeable surface (SPS) if for every $x \in S$, the surface is semipermeable.

Theorem 2.2.1:[20] Let S be a smooth surface in E and $\nu(x) = (\nu_1(x),\ldots,\nu_n(x))$ be its normal vector at each $x \in S$. Then S is a SPS if

(2.2.5) $$\min_{\phi}\max_{\Phi} \sum_{i=1}^{n} \nu_i(x) f_i(x,\phi,\Phi) = 0.$$

proof: $\nu(x)$ is oriented such that it points in the E-direction of S. Let Φ^* be the value of Φ at which the LHS of the equation above assumes its maximum. Then by definition for every feasible ϕ,

(2.2.6) $$0 = \min_{\phi} \sum_{i=1}^{n} \nu_i(x) f_i(x,\phi,\Phi^*) \leq \sum_{i=1}^{n} \nu_i(x) f_i(x,\phi,\Phi^*).$$

This implies there is no value of ϕ that causes penetration of S in the P-direction. The opposite case follows similarly.

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[20] Isaacs pg 9.7

Theorem 2.2.2:[21] Let S be as above and assume that on an open set N relative to S, $\phi^*(x)$ and $\Phi^*(x)$ are $C^{(1)}$ for $x \in S$. Then for k = 1,...,n the following relations hold on N;

$$\frac{dv_K}{dt} = -\sum_{i=1}^{n} v_i(x) \frac{\partial f_i(x,\phi^*,\Phi^*)}{\partial x_K}. \tag{2.2.7}$$

proof: The Tietze-Urysohn extension theorem implies there exists a $C^{(1)}$ extension of $v_i(x)$, $\phi^*(x)$, $\Phi^*(x)$ from N to E. This implies that the partial derivative of (2.2.5) with respect to coordinate x_K, k = 1,...,n exists on the extensions and is given by,

$$\sum_{i=1}^{n} \frac{\partial v_i}{\partial x_K} f_i + \sum_{i=1}^{n} v_i \frac{\partial f_i}{\partial x_K} + \sum_{i=1}^{n}\sum_{j=1}^{s} v_i \frac{\partial f_i}{\partial \phi_j}\frac{\partial \phi^*_j}{\partial x_K} + \sum_{i=1}^{n}\sum_{j=1}^{r} v_i \frac{\partial f_i}{\partial \Phi_j}\frac{\partial \Phi^*_j}{\partial x_K} = 0. \tag{2.2.8}$$

The optimality criterion implies that the two double summation terms are zero. Looking at the first term we may write,

$$\sum_{i=1}^{n}\sum_{j=1}^{s} v_i \frac{\partial f_i}{\partial \phi_j}\frac{\partial \phi^*_j}{\partial x_K} = \sum_{j=1}^{s} \left[\frac{\partial}{\partial \phi_j} \sum_{i=1}^{n} v_i f_i \right] \left[\frac{\partial \phi^*_j}{\partial x_K} \right] = 0. \tag{2.2.9}$$

If $\phi^*(x)$ is an interior minimum of $U(x)$ then the first bracketed term is zero. The alternative case of an exterior minimum implies that the second bracketed term is zero.

[21]Isaacs pg 9.9

Assume that there exists a $C^{(1)}$ real valued function $G:E \to R$ such that $v_i = \partial G/\partial x_i$. The C^1 nature of G implies that $\partial v_K/\partial x_i = \partial v_i/\partial x_K$ since the order of partial differentiation of G makes no difference. Then we may write that

(2.2.10) $$\sum_{i=1}^{n} \frac{\partial v_i}{\partial x_K} f_i = \sum_{i=1}^{n} \frac{\partial v_K}{\partial x_i} f_i = \frac{dv_K}{dt.}$$

The last part of the expression follows from the second summation which is the time derivative of v_K along a trajectory corresponding to the optimal controls Φ^* and ϕ^* through x. Substitution of (2.2.10) and (2.2.9) into (2.2.8) leads to (2.2.7) page 35. This leads to the reverse path equations (RPE) where $\tau = t_f - t$:

(2.2.11) $$\frac{dv_K}{d\tau} = \sum_{i=1}^{n} v_i(x) \frac{\partial f_i(x,\phi^*,\Phi^*)}{\partial x_K}$$

$$\frac{dx_K}{d\tau} = -f_K(x,\phi^*,\Phi^*)$$

<u>**Theorem** 2.2.3:</u>[22] Let *D* be an (n-2)-dimensional manifold in *E* parameterized by $s_1,\dots,s_{n-2}$ with $x_i = h_i(s_1,\dots,s_{n-2})$; i = 1,...,n the smooth coordinate functions of *D*. If the normal vector $\underline{v}(s_1,\dots,s_{n-2}) = \{\underline{v}_1(s_1,\dots,s_{n-2}),\dots,\underline{v}_n(s_1,\dots,s_{n-2})\}$ to *D* is such that on *D*,

(2.2.12) $$\sum_{i=1}^{n} \underline{v}_i(x) f_i(x,\phi^*,\Phi^*) = 0,$$

with $x_i(\tau,s_1,\dots,s_{n-2})$ and $v_i(\tau,s_1,\dots,s_{n-2})$, i = 1,..,n integral curves of the RPE with initial conditions $\underline{v}_i, h_i$, then $x_i(\tau,s_1,\dots,s_{n-2})$ is the parametric representation of a SPS *S* containing *D*.

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[22]Isaacs pg 9.13

proof: We first show that the v_K derived through the RPE is the normal to the surface S. Defining the functions $T_K(\tau,s_1,\ldots,s_{n-2})$, $k = 1,\ldots,n-2$ by,

$$\textbf{(2.2.13)} \qquad T_K = \sum_{i=1}^{n} v_i(x) \frac{\partial x_i}{\partial s_K},$$

we note $T_K(0,s_1,\ldots,s_{n-2}) = 0$ since this is exactly the normality condition on D. Taking the derivative of T_K with respect to τ we have,

$$\textbf{(2.2.14)} \qquad \frac{dT_K}{d\tau} = \sum_{i=1}^{n} v_i(x) \frac{\partial^2 x_i}{\partial\tau\partial s_K} + \sum_{i=1}^{n} \frac{\partial x_i}{\partial s_K} \frac{\partial v_i(x)}{\partial\tau}.$$

Noting that,

$$\textbf{(2.2.15)} \qquad \frac{\partial^2 x_i}{\partial\tau\partial s_K} = -\frac{\partial f_i}{\partial s_K} = -\sum_{j=1}^{n} \frac{\partial f_i}{\partial x_j} \frac{\partial x_j}{\partial s_K} - \sum_{j=1}^{s} \frac{\partial f_i}{\partial \phi_j} \frac{\partial \phi^*_j}{\partial x_K}$$

$$- \sum_{j=1}^{r} \frac{\partial f_i}{\partial \Phi_j} \frac{\partial \Phi^*_j}{\partial x_K}.$$

The optimality criterion implies that the last two terms of the RHS of equation (2.2.15) are once again zero inferring,

$$\textbf{(2.2.16)} \qquad \frac{\partial^2 x_i}{\partial\tau\partial s_K} = -\sum_{j=1}^{n} \frac{\partial f_i}{\partial x_j} \frac{\partial x_j}{\partial s_K}.$$

Substituting (2.2.15) and (2.2.16) into (2.2.14) we conclude,

$$\textbf{(2.2.17)} \qquad \frac{dT_K}{d\tau} = -\sum_{i=1}^{n} \sum_{j=1}^{n} v_i(x) \frac{\partial f_i}{\partial x_j} \frac{\partial x_j}{\partial s_K} + \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial x_i}{\partial s_K} v_j(x) \frac{\partial f_j}{\partial x_i}$$

$$= 0.$$

The fact that $T_K = 0$ on D and (2.2.17) imply $T_K = 0$ on S.

We now define the function $Q(\tau, s_1, \ldots, s_{n-2})$ by,

$$Q = \sum_{i=1}^{n} \nu_i(x) f_i(x, \phi^*, \Phi^*). \tag{2.2.18}$$

Taking the derivative of Q with respect to τ we have,

$$\frac{dQ}{d\tau} = \sum_{i=1}^{n} \nu_i \frac{\partial f_i}{\partial \tau} + \sum_{i=1}^{n} f_i \frac{\partial \nu_i}{\partial \tau} = 0, \tag{2.2.19}$$

by similar arguments to those used earlier. Since Q = 0 on *D*, Q = 0 on *S*.

Let p(w) be a smooth curve on *S* passing thru a point $x \in S$ such that p(0) = x, then the tangent vector components for i = 1,...,n to p(w) at w = 0 are,

$$\sum_{j=1}^{n-2} \frac{\partial x_i}{\partial s_j} \frac{\partial p_j}{\partial w} + f_i \frac{\partial p_\tau}{\partial w}. \tag{2.2.20}$$

The p_τ, p_j are smooth real valued coordinate functions of p(w) such that $s_j = p_j(w)$ and $\tau = p_\tau(w)$. Taking the inner product of the tangent vector of p(w) with the vector ν we have,

$$\begin{aligned}
\sum_{i=1}^{n} \left[\sum_{j=1}^{n-2} \frac{\partial x_i}{\partial s_j} \frac{\partial p_j}{\partial w} + f_i \frac{\partial p_\tau}{\partial w} \right] \nu_i
&= \sum_{j=1}^{n-2} \frac{\partial p_j}{\partial w} \left[\sum_{i=1}^{n} \frac{\partial x_i}{\partial s_j} \nu_i \right] + \frac{\partial p_\tau}{\partial w} \left[\sum_{i=1}^{n} f_i \nu_i \right] \\
&= \sum_{j=1}^{n-2} \frac{\partial p_j}{\partial w} \left[T_j \right] + \frac{\partial p_\tau}{\partial w} \left[Q \right] \\
&= 0.
\end{aligned} \tag{2.2.21}$$

This implies that the vector v is normal to the surface S. Lastly S is semipermeable because,

$$(2.2.22) \quad \min_{\phi} \max_{\Phi} \sum_{i=1}^{n} v_i(x) f_i(x,\phi,\Phi) = \sum_{i=1}^{n} v_i(x) f_i(x,\phi^*,\Phi^*)$$

$$= Q = 0.$$

2.2.4 BARRIERS

Clearly the barrier B that separates the capture and escape region of E is a semipermeable surface. The barriers B that we will be interested in will be attachable to the terminal manifold C. This corresponds to the case where the Barrier is a function of the maximum vehicle dynamics. It is possible for a barrier to exist that does not attach to C. The case of an intercept problem where fuel is limited is an example of such a case. Assuming the pursuer is faster, fuel limited, and equally maneuverable as the evader, it is obvious that a barrier exists corresponding to fuel exhaustion that prevents capture. We will not be concerned with these type of barriers here, as traditional separation principles allow us to deal with these type of constraints.

The boundary of the usable part (BUP) has been shown to be an (n-2)-dimensional manifold such that BUP = all $x \in C$ such that,

$$(2.2.23) \quad \min_{\phi} \max_{\Phi} \sum_{i=1}^{n} \underline{v}_i(x) f_i(x,\phi,\Phi) = 0,$$

where $\underline{v}_i(x)$ is the normal to $x \in C$ pointing into E. The $x \in$ BUP and $\underline{v}$, serve as the initial conditions to the RPE,

$$\begin{aligned} \frac{d\nu_K}{d\tau} &= \sum_{i=1}^{n} \nu_i(x) \frac{\partial f_i(x,\phi^*,\Phi^*)}{\partial x_k}, \\ \frac{dx_k}{d\tau} &= - f_k(x,\phi^*,\Phi^*), \end{aligned} \tag{2.2.24}$$

where ϕ^* and Φ^* are the continuous solutions on E to

$$\min_{\phi} \max_{\Phi} \sum_{i=1}^{n} \nu_i(x) f_i(x,\phi,\Phi) = 0.$$

The theorem of the last section implies then that the (n-1)-dimensional manifold generated by the RPE and initial conditions on C is a semipermeable surface.

There are a number of important observations about the geometry of the barriers. The first is that they may end abruptly when a solution no longer exists to (2.2.24) for the controls, or at a curve of discontinuity of the optimal controls.

The BUP may not be connected (actually the most common case). This means that there are a number of barriers connected to the terminal manifold. Each of these barriers is attached to a connected submanifold of dimension (n-2) of BUP. The way these barriers intersect determines the capture and escape regions. If the barriers and UP of C form the boundaries of closed regions of E then the interior of these regions are the capture region if the optimal controls agree for each barrier at the intersection point. It may be that the capture region may not be connected, but consists of a number of connected subregions. The escape region consists of that part of E that is not part of the capture region or its boundaries. The boundary between these two regions of course consists of portions of the barriers.

The barriers may in fact not meet or form the boundary of a closed region of E. In this case, assuming no other type of barriers not connected to C, the playing space E serves as the capture region. The barriers become a singular surface that trajectories must swerve around to reach the UP of C.

3. SELF MODIFYING SYSTEMS

This section explores systems and modelling techniques that fall outside the classical techniques. Mesarovic formalized the traditional concepts for systems in which there was a fixed set of objects, and a fixed set of relations between the objects. The type of systems we are exploring have the property that the system is self modifying in the objects and relations. These type of systems we term semantic systems. It is important to understand how a system such as air-combat can evolve before an attempt is made to the control the system's evolution. Semantic systems will be shown to be a natural extension of Mesarovic's systems. Simulation techniques for investigating the complexity of semantic systems will also be explored.

3.1. GENERAL SYSTEMS

A "system" can loosely be defined as an aggregation or assemblage of objects joined in some regular interaction or interdependence. A system "entity" is an object of interest. Properties of "entities" are "attributes". "Activities" are processes that cause changes in the "system". We start by defining the traditional mathematically rigorous definition of a general system as originated by Mesarovic (5).

Definition: Let T be an arbitrary linear ordered index set, a formal object $\underline{\underline{E}}_j$; $j \in \{1,2,...,n\}$, is a set whose elements $e^j(t)$; $t \in T$, are called the values of the object $\underline{\underline{E}}_j$.

Note: The index set T may be of finite, countable or uncountable cardinality. The case of Card(T) = 1 might correspond to the constrained extremization system, while card(T) = R and card (T) = N would correspond to continuous and discrete time systems respectively. We now define general systems and the more structured case of input-output systems.

Definition: (Explicit) A general System E_s is generated by the Cartesian cross product:

$$\underline{\underline{E}} = \underline{\underline{E}}_1 \times \underline{\underline{E}}_2 \times \ldots \times \underline{\underline{E}}n$$

and the relation $\underline{\underline{R}}$ on $\underline{\underline{E}}$ such that E_s is a proper subset of $\underline{\underline{E}}$ (i.e. $e(t) \in Es$ iff $\underline{\underline{R}}[e^1(t),e^2(t),\ldots,e^n(t)]$).

<u>Definition:</u> Let I = {1,2,...,n} and I_X, I_Y be sets forming a partition of I. The set X = x{$\underline{\underline{E}}$j: j ∈ I_X}[23] is termed the input object, while the set Y = x{$\underline{\underline{E}}$j: j ∈ I_Y} is termed the output object. The system E_S is then

$$E_S \subset X \times Y$$

and is referred to as an input-output system.

Input-output systems are defined in terms of relations. This means that there may be a set of outputs for a given input. If we apply a particular input to an actual system we know we will get out a single output, not a set of outputs. This means that there is something still missing from our definition: the concept of state that determines what the output should be for a particular input.

<u>Definition:</u> Given a general system E_S, let C be an arbitrary set and F a function, F:(C x X) → Y, such that (x,y) ∈ Es iff there exist c ∈ C such that F(c,x) = y. C is then a global state set whose elements are the global states. F is a global systems-response function.

It should be noted that a state object is not equivalent to having a state space. A state space exists only for dynamical systems in which the semigroup property (among others) is in effect. Dynamical systems are a specific type of family of general systems.

The problem with this explicit specification of a general system is that both $\underline{\underline{R}}$ and $\underline{\underline{E}}$ may be infinite sets which may be only specified through induction. Thus $\underline{\underline{E_j}}$ may be defined by an initial set of generating elements E and a set of syntactic rules (S) to generate syntactically correct elements from previously generated

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[23]Cartesian product

elements. The system relations $\underline{\underline{R}}$ may also specified inductively. A set of primary relations (R) is given, and a set of axioms (A) is specified that defines how members of R are combined to form $\underline{\underline{R}}$. This leads to the implicit definition of a system.

Definition: (Implicit) A general system is a quadruple (E,S,T,A).

We will now introduce the formal concept of a classical system model as defined by Mesarovic (5). Conceptually a model should be a system that simplifies the system being modelled such that "important" properties are incorporated while other "unimportant" properties are neglected.

Definition: Let $Es \subset X \times Y$, $Es' \subset X' \times Y'$ be two general systems, with the functions $h_X: X \rightarrow X'$, $h_Y: Y \rightarrow Y'$. The function $h = h_X \times h_Y: X \times Y \rightarrow X' \times Y'$ is called a relational homomorphism from Es to Es' if and only if:

$$(x,y) \in Es \rightarrow (h_X(x), h_Y(y)) \in Es'$$

if and only if h is a bijective relational homomorphism such that:

$$(x',y') \in Es' \rightarrow (h_X^{-1}(x), h_Y^{-1}(y)) \in Es$$

then h is a relational isomorphism.

Es' is a structural model for Es if and only if there exists a relational homomorphism $h: Es \rightarrow Es'$ and the two systems are structurally equivalent if and only if h is a relational isomorphism.

A general system model is then an quintuple (E,S,T,A,h).

3.2. SEMANTIC SYSTEMS

The paradigm for analysis and synthesis of systems in the past have assumed Mesarovic's definition of systems and models.[24] There are many complex systems where this definition falls short. Many complex systems are self modifying. This modification takes place in an evolution of the generating relation $\underline{\underline{R}}$, the system objects $\underline{\underline{E}}_j$, or the order of the system n. Air combat is one system that exhibits these properties. The system components consist of aircraft, missiles, SAMS, and ground targets. The destruction of an object or launching of missiles modifies the systems order and relations.

Let $E_S{}^i$ be a Mesarovic general system defined by the relation $\underline{R}^i$ such that $E_S{}^i \subset \times\{E_j{}^i\colon j \in J_i\}$. The set $E = \{E_S{}^i\colon i \in I\}$ is the set of system generation components. This set defines the types of feasible components with which we are concerned, and multiple copies may exist. The set E for the air combat problem would consist of airplanes, missiles, SAMS, and ground targets.

We now go about defining the feasible partial relations that coordinate components of the system. A set of feasible relations $\mathcal{R}^i$, for each $i \in I$ is defined. The domain for each $r \in \mathcal{R}^i$ is of the form $\times\{E_j{}^{Ka}\colon j \in J_{Ka}, J_{Ka} \subset J_K, K_a \in I, a \in A\}$. The range for each $r \in \mathcal{R}^i$ is of the form $\times\{E_j{}^i\colon j \in J, J \subset J_i\}$. The index set A defines the mix of components that effects a component of type i, K_a indicates the type of component for $a \in A$, and J_{Ka} indicates what objects from a component of type K_a effect a component of type i. The index set J indicates what objects from a component of type i are effected. The case of input-output systems has the domain restricted to output objects, and the range restricted to

[24]The practitioner may not have been cognizant of this fact.

input objects. It should be noted that a component of type i may be effected by multiple components of type j though the object sets from the multiple components of type j may be different. The set $\mathcal{R} = \cup_{i \in I} \mathcal{R}^i$ defines the set of feasible relations for the system.

A semantic state consists of the tuple (S, R_S, M) where

$$S = \{E_S^{i_m} : i_m \in I,\ m \in M\},$$

$$R_S = \{r^m : i_m \in I,\ \text{type}(r^m) \in \mathcal{R}^{i_m},\ m \in M\},$$

and M defines the mix of components for the system. The notation r^m is used to indicate a feasible relation for a component of type $i_m \in I$ from $\mathcal{R}^{i_m}$ for $m \in M$. We note that two relations for component m, r^m and r^m may be of the same type but are considered different if the components assigned to the domain are assigned differently or are different. We call M the semantic order, S the semantic form, and R_S the relational state. The semantic form S, is fixed for a fixed M, I.E. a component may not change its type during the course of its existence. Alternatively, R_S is not fixed for fixed M, a component may change its relation to other components as the need arises. A change in (S, R_S, M) is termed a semantic event.

The semantic order M may evolve in two manners, reproduction or attrition. Letting M' denote the new semantic order, then $M \subset M'$ indicates a pure reproductive semantic event. Similarly, $M \supset M'$ indicates a pure attrition semantic event. A semantic event may occur such that $M = M'$, implying that only the relation state has changed. This type of semantic event is termed a relational semantic event. The classification of a semantic event as pure is not guaranteed, an event of mixed origin is the normative case.

The relational state R_S is also subject to change. Obviously a change in semantic order will force a change in R_S. The relational state may also change while the semantic order remains fixed. This change would take place *dependant upon the traditional global state and systems-response* of the component types indicated by the semantic form S. The purpose of S is to carry the traditional Mesarovic information.

The values of the objects of the components of S may be varying depending upon classification (I.E. dynamical, decision) of the general system type of the component. R_S coordinates the components of S, hence R_S has an influence on the components of S. S may be continuously changing with respect to object values of the components, but the member components only change at semantic events.

We will now describe the constraints under which a semantic system can evolve. Each component type Es^i, $i \in I$, has a set of reproductive rules F_i that may be empty. Members of F_i consist of the ordered tuples $(Es^j, \mathcal{R}_j, C_R^j)$ such that $j \in I$, $Es^j \in E$, $\mathcal{R}_j \subset \mathcal{R}^j$, and $C_R^j \subset Es^i$. The tuple indicates that the component of type Es^i may reproduce a component of type Es^j with an initial relation $r \in \mathcal{R}_j$ if the conditions for reproduction C_R^j are met by component of type Es^i. The set $\mathcal{R}_j$ indicates the set of feasible relations from which the new component can be initialized. The set $\mathcal{R}_j$ may not be empty. The set C_R^j indicates the conditions on the component of type Es^i under which the new component may be reproduced. Clearly if C_R^j is empty then reproduction of the new component is not possible, $C_R^j = Es^i$ implies that reproduction is always possible.

We have so far described the type of reproduction that may take place for a member of the system generation component set. We now describe the consistency conditions that express a component can not reproduce another component if there does not exist a initial feasible relation for the new component whose domain consists of component types currently in the system. A component $m \in M$ of type i_m may reproduce a component of type Es^j when $(Es^j, R_j, C_R^j) \in F_{im}$ only if there exists subset $\underline{M} \subset M$ with $m \notin \underline{M}$, $r \in R_j$, and a correspondence $C{:}A \rightarrow \underline{M}$ such that $Ka = i_C(a)$ for every $a \in A$.[25] This simply states that reproduction can only take place when there exists an initial feasible relation whose domain can be well defined by an appropriate assignment of components currently in the system.

The attrition rules for a semantic system describe how components of M may be removed from the system. There are two classifications of attrition rules for semantic systems. The first classification consists of the rules of self attrition which describe the mechanism by which a component can cause itself to be removed from M. The self attrition rules for a component of type Es^i are defined by the set of self attrition rules $C_{SA} \subset Es^i$. C_{SA} specifies the conditions under which a component ceases to exist independent of the action of all other components of the system. We note that C_{SA} must be a proper subset or the component could not possibly exist. It is also possible that C_{SA} is empty. Self attrition is closely related to concepts such as stability, and life expectancy. An aircraft that goes unstable will cease to exist once it hits the ground. A missile has a life expectancy related to the burn time of its motor.

The other classification for attrition in a semantic system is attrition of one component caused by another. This type of attrition is termed causative attrition. Causative attrition is defined for each component type Es^i, $i \in I$ by the set of rules

[25] Ka and A were defined previously for the definition of the domain of a feasible relation.

G_i which may be empty. A member of G_i is the ordered tuple $(Es^j, C_{CA}{}^j)$ such that $j \in I$, $Es^j \in E$, and $C_{CA}{}^j \subset Es^i \times Es^j$. The tuple indicates that the component of type Es^i will cause a component of type Es^j to be removed if the conditions for attrition $C_{CA}{}^j$ are met by Es^i and Es^j. $C_{CA}{}^j$ must be a proper subset to have significant meaning. $C_{CA}{}^j$ empty implies that a component of type Es^j may not be removed by a component of type Es^i. The case $C_{CA}{}^j = Es^i \times Es^j$ would express that the existence of a component of type Es^i implies the removal of a component of type Es^j. An important point of semantic systems is that it may not be possible for a component of type Es^i to remove a component of type Es^j directly, but a component of type Es^i may be able to reproduce a component of type Es^K that can remove the component of type Es^j.

3.3. SEMANTIC CONTROL

Previously we have given a description of self modifying systems that have been termed semantic systems. A component of type E_S^i ($i \in I$) of a semantic system had the capability to choose a relationship $r \in \mathcal{R}^i$ to other components. A component also had the capability to reproduce other components according to the set $\mathcal{F}_i$ or cause attrition of other components according to the set $\mathcal{G}_i$. The choice of a components relation to other components, to reproduce a component, or remove a component is a form of control over the semantic state of the system. This type of control is termed semantic control.

Each component or group of components is assumed to have a directive. The directives specify the purpose of a component. Groups of components with the same directive may cooperate. These directives for the air combat problem correspond to missions. A group of fighter bombers for the air combat problem may be assigned the mission of destroying a tactical site, they may cooperate in achieving this directive either in bombing runs or handling interceptors.

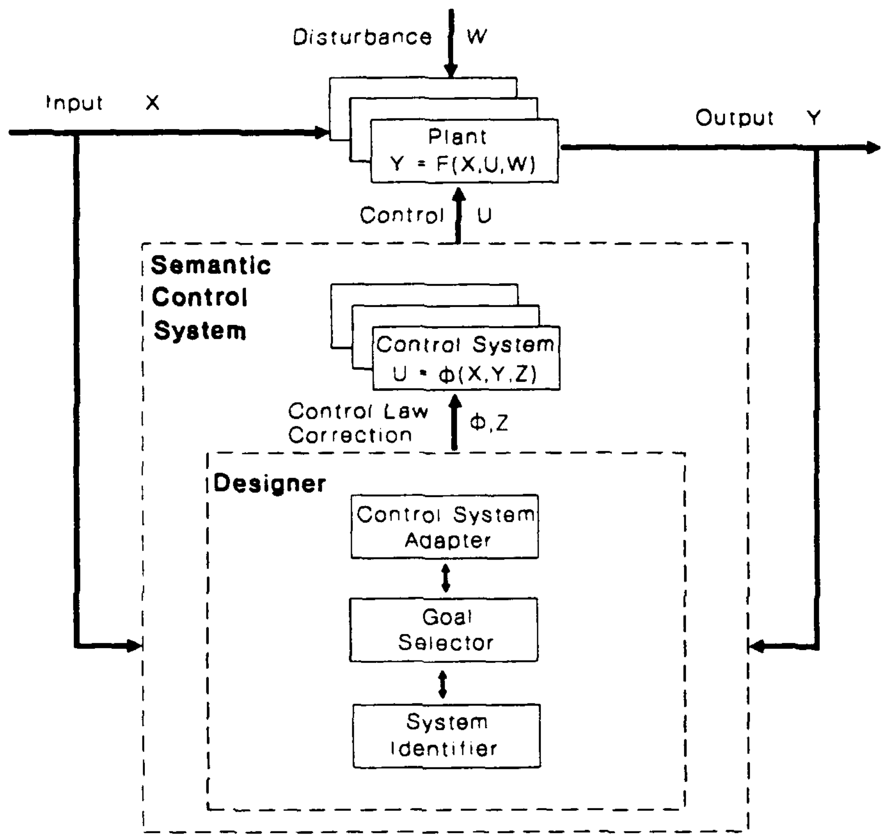
The semantic control problem for a component $i \in I$ is to determine the "best" $r \in \mathcal{R}^i$ dependant upon the semantic order M that allows the component its "best" attempt to achieve its directive. It should be noted that a component may only be able to achieve its directive by reproducing another component. The "best" $r \in \mathcal{R}^i$ in this case is one that brings the component to a condition of $\mathcal{F}_i$ for reproduction. This means for the air combat problem an aircraft chooses a strategy ($r \in \mathcal{R}^i$) that brings the aircraft to a missile firing point with its target.

The set of all semantic states (S, R_S, M) clearly can be an uncountable set. A component $m \in M$ may also only have limited or no knowledge about other $m' \in M$ (stealthing, limited sensor range). Clearly, it is not possible to anticipate every situation or even to determine if a specific situation exists. Still a component must try to meet its directive as best it can with limited knowledge and understanding of specific semantic states. The difficulty of designing a semantic controller is one reason humans are still in the loop in many real world systems that are in reality semantic systems.

Semantic control contrasts in many essential details with classical control. Classical control assumes that the designer can analyze the plant and generate the control laws a priori. The underlying assumption is that plant and control laws are fixed. This implies that the plant is a Mesarovic type system in which the set of formal objects $\underline{\underline{E}}_j$; $j \in \{1,2,..,n\}$ is fixed and that the generating relation $\underline{\underline{R}}$ is also fixed.

The semantic control system consists of a set of classical control systems which represent choices for $r \in \mathcal{R}^{im}$ for components $m \in M$ under control of the semantic control system, along with a designer as depicted in figure 3.1.

FIGURE 3.1
SEMANTIC CONTROL SYSTEM



The designer may be automated or semi-automated and consists of three blocks:

1. System Identifier,
2. Goal Selector,
3. Control System Adapter.

These blocks are generically referred to as correlators. Each correlator searches its specific search space for the best alternative according to the current data and specific search knowledge.

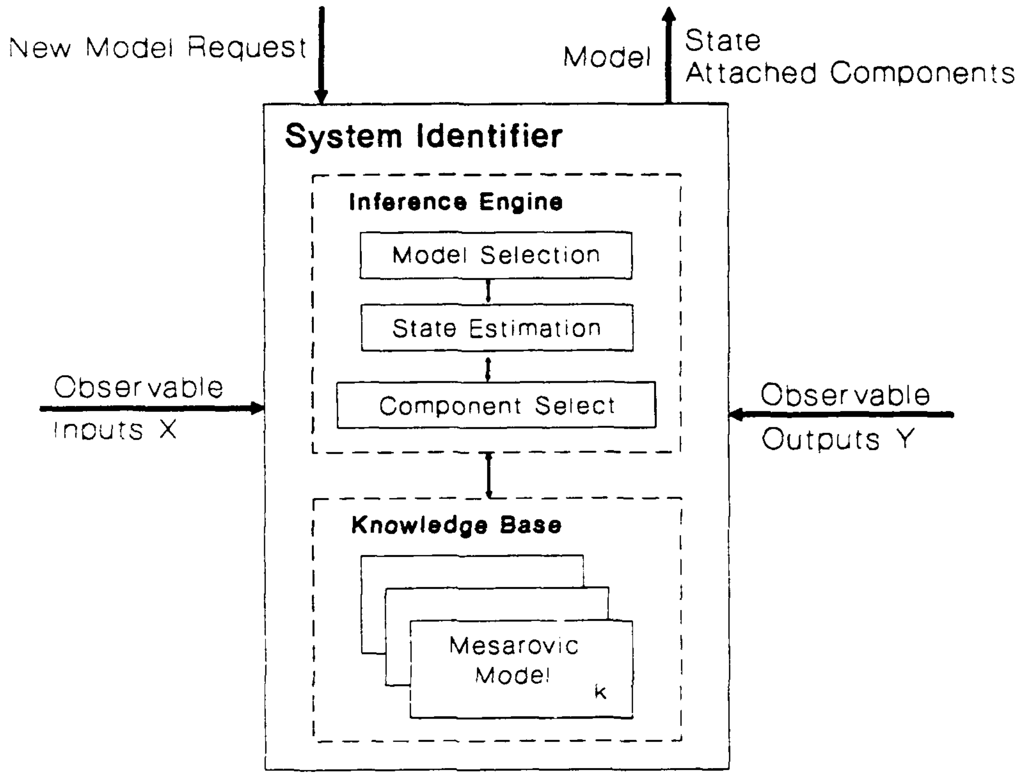
The purpose of the System Identifier correlator is to estimate the current semantic state. *The identifier correlator consists of an inference engine and knowledge base.* The knowledge base consists of a set of structural models (I_M)

of Mesarovic form. This set of structural models may be explicitly defined, or implicitly defined via a generating set and an algebra of construction. Typically a member of I_M has subsystems that are structural models for components of *E*.

The inference engine for the System Identifier must choose the best structural model from the knowledge base and estimate the state object from the state object set for the chosen structural model. There may be more components in the semantic order M than any of the structural models in the knowledge base can account for. The inference engine must choose which of the components of M will be accounted for in the structural model chosen.

The System Identifier for the air combat problem for a specific aircraft component determines how the aircraft component itself should be modeled along with enemy aircraft. Assuming the knowledge base can only handle one-on-one encounters the inference engine might choose to model the aircraft itself as an infinitely maneuverable fixed velocity two dimension model. An adversary might be modeled as a limited maneuverable but faster two dimension model. The identifier would also have to determine best approximations of the state of this lower dimension model that approximate the much higher order aircraft components.

FIGURE 3.2

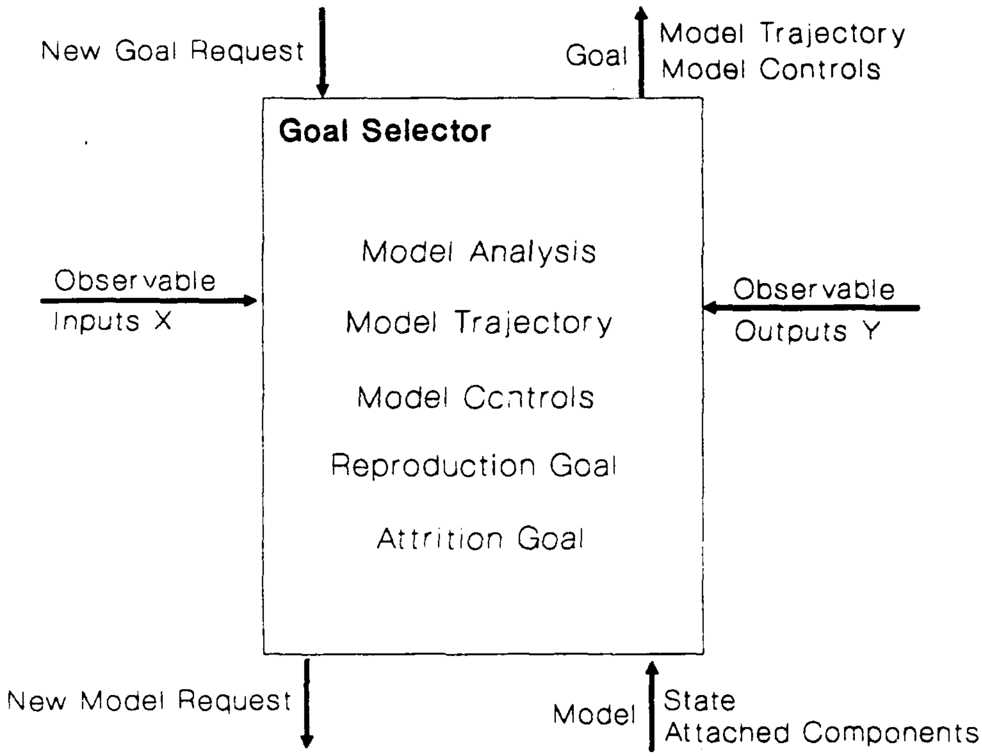


SYSTEM IDENTIFIER

The Goal Selector correlator determines a trajectory in the semantic state space based on the structural model identified by the identifier. This trajectory is chosen to allow survivability and reliability according to the component's directive. It may not be possible to meet this semantic goal exactly.

The Goal Selector for the air combat problem solves the differential game specified by the System Identifier correlator. The results are the optimal trajectories, barriers and controls. These controls are not the same controls as those needed by the aircraft, due to the fact that the model chosen by the Svstem Identifier is a simplification of the actual encounter.

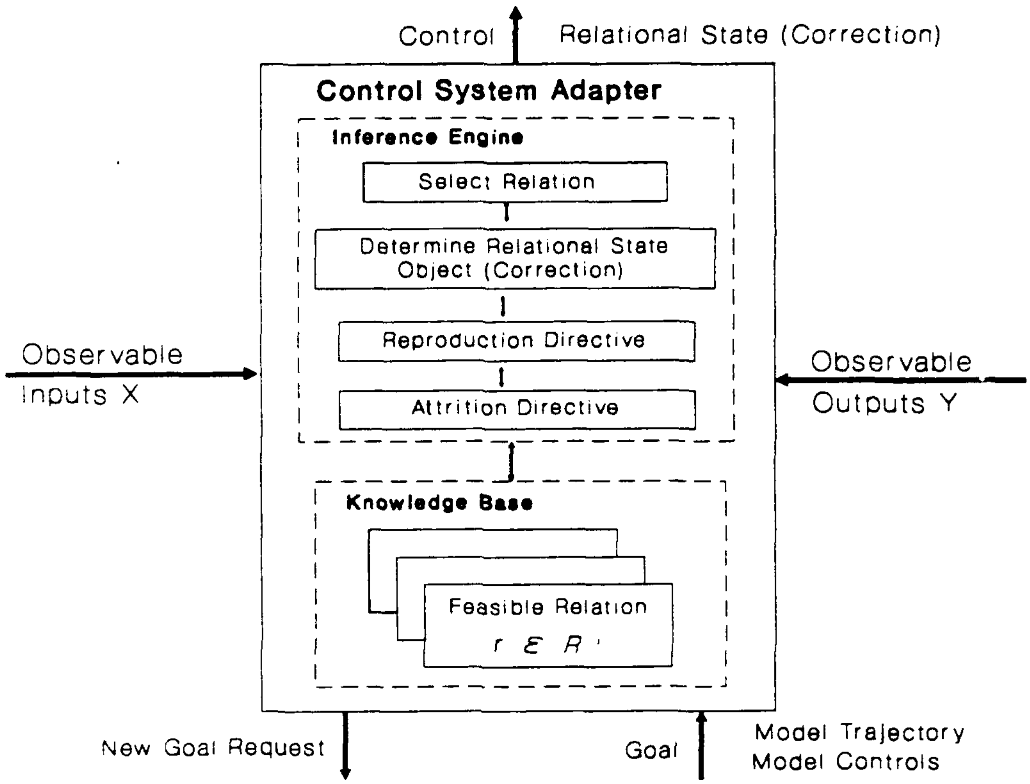
FIGURE 3.3



Goal Selector

The Control System Adapter determines the control strategy that will meet the semantic goal. This control may be of the open/closed loop variety, or may be rule based, or a combination of all three. The control system adapter chooses its control for a component of type $E_S{}^i$ from the set of feasible controls R^i. The set of feasible controls R^i then serves as the knowledge base for the Control System Adapter. The inference engine chooses the member of the knowledge base that meets the goal chosen by the goal selector as close as possible. The control adapter also determines if reproduction or attrition goals can be met and carries out this directive.

FIGURE 3.4



Control System Adapter

The Control System Adapter for the air combat problem determines the aircraft controls that causes the aircraft to "best" follow the optimal trajectory determined by the Goal Selector. This is accomplished by a rule-based control system dependant upon the differential game model chosen in the System Identifier, and upon the performance limits of the aircraft.

Lastly, each correlator may feed back information to another correlator whenever it cannot meet the performance criteria asked for by the other correlators.

3.4. FRAME BASED SIMULATION OF SEMANTIC SYSTEMS

The majority of large scale systems that are under study in today's technological world are of such complexity that closed form analytical tools provide only a fraction of the information needed to understand and design these systems. The paradigms of analysis assume a homogeneous form of a system. Classes of these paradigms deal with systems that are entirely discrete dynamical systems, continuous dynamical systems, decision making systems, linguistic systems, etc. Real world systems are of such high order and mixed type that simulation techniques are a must to understand these systems.

Semantic Systems have components that have the complexity described above. Another order of complexity is added in the self modifying nature of Semantic Systems. Stability and controllability of certain components may be desirable sub-properties, but more important design goals might be reliability and survivability of certain groups of components. This is indeed the case for air-combat problems in which the designer desires survivability and reliability only on his components. Simulation currently is the only method for design and analysis for these systems.

How do we go about modelling and simulating semantic systems in which the existence and mix of objects can be considered part of the state of the system? The paradigm of Frame Base Simulation (FBSM) is proposed as a methodology for dealing with Semantic Systems and other highly complex general systems.

Minsky (1975) proposed the idea of frames and scripts to represent knowledge about objects and events typical to specific situations for artificial intelligence (AI) knowledge representation. This is a method of organizing the knowledge representation in a way that directs attention and facilitates recall and inference. Minsky's frames were based on "declarative" knowledge. Knowledge could be separated into a general set of procedures for manipulating facts of all types, and a set of facts describing a particular area of knowledge. Mathematically this is akin to having a set of proof procedures that are applied to a specific group of axioms. Winograd asserted that though possible, the restriction of frames to declarative knowledge is overly cumbersome. Winograd added the idea of attached procedures to Minsky's frames. These attached procedures represented "procedural" knowledge. "Procedural" knowledge is knowledge bound to the use of the knowledge, or programs associated with how to use the knowledge.

Frames from the AI standpoint provide the structure that allows interpretation of new data via the concepts accumulated by previous experience. This format for AI facilitates "expectation-driven processing", looking for things based on the conjectured context. An example of a generic chair frame might be:[26]

CHAIR Frame

Specialization-of:	Furniture
Number-Of-Legs:	an integer (Default =4)
Style-Of-Back:	straight, cushioned, ...
Number-Of-Arms:	0, 1, or 2

Specialization-of, Number-Of-Legs, Style-Of-Back, and Number-Of-Arms are "slots" of the generic chair frame. Slots are the elementary representational

[26]Barr page 217

mechanism of "expectation-driven processing". Slots provide the place where knowledge fits within the larger context created by the frame. An important property of frames is inheritance. The Specialization-of slot establishes the "property inheritance hierarchy" between the frames. Descendant frames can inherit information of parent frames via this mechanism. A particular instantiation of chair inherits the same slots from a generic chair frame, but the slots are more fully specified:

Roark's-CHAIR Frame

Specialization-of:	CHAIR
Number-Of-Legs:	1[27]
Style-Of-Back:	cushioned
Number-Of-Arms:	2

We want to model a semantic system in which the semantic state is defined by (S, R_S, M). We start with the set $E = \{E_S^i : i \in I\}$ of system generation components.[28] The set E for the air combat example consists of aircraft, tactical sites, ground defenses (SAMS) , air to air missiles, ground to air missiles, air to ground missiles. Each $E_S^i \in E$ is described by a structural model $E_S'^i$ and a generic FRAME.

Frames from the modeling standpoint provide the structure in which the evolution of the system can be viewed. This is "context-driven modeling", the component is modeled based on the context. The "slot" again serves as the elementary representational mechanism. A slot for FBSM may contain in

[27] I sit on an executive office chair with a pedestal base due to my many hours sitting in front of my computer

general, subslots, attached procedures, and other frames as is the case for AI. An abridged generic aircraft frame would be:

Aircraft *Frame*

- Mesarovic Slot
    - Parameters:
        - Max Clift: a real
        - Max Thrust: a real
        - Max G's: a real
        - Weight: a real
        - Wing Surface Area: a real
        - Oswald Efficiency: a real
        - Wing Aspect Ratio: a real
    - State:
        - X: a real
        - Y: a real
        - Z: a real
        - V_{LG}: a real
        - A_{LG}: a real
        - Heading Rate: a real
        - Pitch Rate: a real
        - Heading: [-Pi,Pi]
        - Pitch: (-Pi/2,Pi/2)
        - Fuel: a real
        - Number_Air_Air_Missiles: [0,1,2,3,4,5,6]
    - *Controls:*
        - Bank: (-Pi/2,Pi/2)
        - Thrust: (0, Max_Thrust)
        - Clift: (0, Max_Clift)
    - *if-needed* (procedure to update state)
- Relational State Slot
    - Tactic: (LOS_Pursuit, LOS_Evade, LOS_Ground_Pursuit, LOS_Ground_Evade Harrier_Attack_F16 F16_Evade_Harrier)
- Reproductive Slot
    - Air To Air Missiles: (Aim_9J, Aim_9L)
        - Missile Description:
            - Max Off Boresight: [0,Pi]
            - Min Range: a real
            - Max Range: a real
        - *if-needed* (procedure to fire missiles)
    - Air To Ground Missiles: (****, ****)
        - ************************************
        - ************************************
- Self Attrition Slot
    - if-needed (procedure to test for crash)

---

[28]Recall that each member E_S^i of E is a Mesarovic general system.

A generic frame is defined for each $i \in I$. Each generic frame contains slots for modelling the component i. The first type of slot defined is the "Mesarovic Slot". This slot contains the Mesarovic structure of $E_S'^i$, a structural model of E_S^i. The declarative portion of the Mesarovic slot contains subslots defining the global state set, along with component input and output object ($E_S'^i_j$) sets of $E_S'^i$. A global state is defined when global state subslots are filled. The procedural portion of the Mesarovic slot contains attached procedures representing the global systems-response function for $E_S'^i$. These attached procedures indicate how to determine the output set subslot values, and state subslot values (for the dynamical portion of the system) from input subslot values.

The state object slots for the generic aircraft frame consist of the subslots,

"Parameters" x "State"

We note that we have been able to partition the global state set into a constant portion and a changing portion. The "State" portion of the partition corresponds to the segment of the general system for which the semi-group property holds. Mesarovic has shown that under this property the more restrictive notion of state space can be defined along with the corresponding transfer function. The input object set slots consist of the "Control" slots. The output object set slots are not included in this example as they are redundant with the State slots for an aircraft model. The attached procedure representing the aircraft transfer function consists of the discrete time version of the aerodynamic equations of motion.

The second type of slot defined for each generic frame is the "Relational State Slot". This slot contains frames indicating the feasible relation $r \in \mathcal{R}^i$ currently representing the relational state of the component i. This is a example of when a frame slot may contain another frame. A relational frame is defined for every $r \in R^i$. The relational frames will be described later but an example for the air combat problem is shown below.

Harrier Attack of F16 Frame	
Harrier Description:	
Speed:	a real
Kill Radii:	a real
Kill Eccentricity:	a real
F16 Type Target:	an aircraft frame
F16 Description:	
Speed:	a real
Kill Radii:	a real
Kill Eccentricity:	a real
Max Turn Radius:	a real
if-needed (procedure to determine Harrier control)	

The next slot of interest is the "Reproductive Slot". This slot contains subslots defining the type of components that may be reproduced ($E_S{}^j$), the initial state object and relation (R_j) for a reproduced component, and conditions under which reproduction may take place ($C_R{}^j$). The declarative portion of the slot contains a subslot whose value is the frame type of the object to be *reproduced*. The condition for reproduction and initial relation are determined by both declarative and procedural subslots. The parameterization of the reproductive conditions ($C_R{}^j$) may be specified by declarative slots, while the procedure for testing for ($C_R{}^j$) is done via an attached procedure.

The reproductive slot for the generic aircraft frame contains the type of missiles that may be fired (reproduced). The type of air to air missile subslot includes a frame subslot containing a specialization of an air to air missile frame (Aim_9L, Aim_9J). There are also declarative subslots describing the capabilities of a missile. These conditions are tested by the attached procedure to determine if a missile should be fired.

Slots are also defined for attrition of both types. The self attrition slot contains an attached procedure to determine if the conditions C_{SA} have been met. These conditions for the aircraft frame include an attached procedure for testing for out of fuel, flying into the ground, or exceeding the design of the air frame.

The causative attrition slot contains attached procedures for determining if the conditions $C_{CA}{}^{j}$ are met for removing a frame instantiation of type j. This type of slot is not present for the aircraft frame assuming no guns.

We now turn our attention to the relational frames. A generic relational frame is specified for each $r \in R^i$. The declarative portion of the frame carries the object state set for the relation. This set carries the paramterization of how the relation is to be fitted to the component frame to which it is attached. The declarative portion also contains slots containing frames that contain instantiation of the components that serve as the domain of the relation. The procedural portion contains an attached procedure to determine the inputs of component i based on the declarative portion cf the slot (domain components and relational state). An instantiation of a relational frame occurs when a component i chooses relation $r \in R^i$.

The "Harrier AttacK of F16" frame contains a slot for a specific instantiation of an F16 frame indicating the current target for the Harrier instantiation. The description slots describe a structural model for both the F16 and Harrier instantiations. These models are of much lower order than the actual structural models specified in the Mesarovic slot of the component frame. These lower order models are used so that an actual solution to the relation can be computed.

The structure and management of frame instantiation is what separates FBSM from simply being a data structure. Inheritance allows one frame to be related to another. The generic aircraft frame was defined above. We may further specify aircraft types by the use of inheritance. An F16 type aircraft would have the same slots as the generic aircraft frame but the Parameter slots would be specified. Similarly to the AI case an F16 would be a specialization of the aircraft frame. The same applies to missiles where we have used the specialization of AIM_9L and AIM_9J. A specialization of a frame is different than an instantiation. A instantiation of a frame implies the existence of a system component. A F16 is a type of aircraft but Blue Fighter 1 which is an instantiation of a F16 is an actual component. Reproduction is the act of instantiation of a specialization of a frame. The case where Blue Fighter 1 (F16) fires an AIM_9L missile means that the Blue Fighter 1 will cause the instantiation of a missile that inherits the properties of an AIM_9L missile frame.

Frames from an AI standpoint, by supplying a place for knowledge, create the possibility of missing or incompletely specified knowledge. The slot mechanism permits reasoning based on seeking confirmation of expectations (I.E.. fill in the blanks). The use of attached procedures can be used to determine the values. For modelling, initially we assume all slots are filled. Subsequent slot values and instantiations are determined with the use of attached procedures and allow for FRAME interdependencies. An FBSM executive is responsible for the management of the simulation system. This executive keeps track of instantiations, removes instantiations, and manages iteration events for slot values. The FBSM of the air combat environment has been implemented in PASCAL and currently runs on a SUN 4.

4. AIR COMBAT ENVIRONMENT SIMULATION

The air combat environment simulator will now be described. The simulator will be described in terms of Semantic Systems and Frame Based Simulation (FBSM) described in Section (3).

The index set I of system generation component set E consists of Aircraft, Ground Targets, and Ground Defenses. We note that in our stage one simulator we are not including air to air missiles, air to ground missiles, and ground to air missiles. One additional advantage to FBSM is that a model of the causative attrition rules of a component may be absorbed into the Frame model of the components that can reproduce the component.

The section will be organized in terms of the component types of the index set I. Each component type will have a section describing the structural model E'^{i}_{S} for $E^{i}_{S} \in E$, followed by Frame description. The first component we will consider will be aircraft.

4.1. AIRCRAFT

This section of the simulator description will be devoted to the aircraft components. We first will discuss the aerodynamic model for an aircraft, a continuous time dynamical model. The discrete time numerical model for the simulation will then follow. The discussion will then continue with a probabilistic model for the missile capabilities of the aircraft. Lastly we will define the aircraft component frame.

4.1.1. AERODYNAMIC MODEL

There are four aerodynamic forces that must be considered, gravity, thrust, lift, and drag. The aircraft frame has three axis: longitudinal, lateral, and vertical. The longitudinal axis is through the aircraft body. The lateral axis is through the aircraft wings perpendicular to the longitudinal axis. The vertical axis is perpendicular to the plane formed by the longitudinal and lateral axis.

The following angles define the aircraft movement in three dimensional euclidian coordinates: θ, ϕ, and τ. The heading of the aircraft in the x-y plane is defined by θ. This angle is measured counterclockwise from the x-axis.

The rotation of the aircraft about its lateral axis is defined by the pitch angle ϕ. The pitch angle is measured clockwise from the horizontal x-y plane. Pitch angle defines if the aircraft is flying nose up or down with respect to the horizontal plane.

The last angle τ defines the rotation of the aircraft about its longitudinal axis. The bank angle τ is measured clockwise from the x-y plane. Bank angle defines the aspect of the wings with respect to the x-y plane.

Thrust is the force applied by the aircraft engine along the forward longitudinal axis of the aircraft. If T is the magnitude of the thrust (NT), then the components are,

$$
\begin{aligned}
T_X &= T\cos\phi\cos\theta \\
T_Y &= T\cos\phi\sin\theta \qquad \textbf{(4.1.1)} \\
T_Z &= -T\sin\phi
\end{aligned}
$$

The lift force is directed along the vertical axis of the aircraft. Lift is the force induced by the wings of the aircraft. If L is the magnitude of the lift its components are:

$$L_X = L(\sin\phi\cos\theta\cos\tau + \sin\theta\sin\tau)$$
$$L_Y = L(\sin\phi\sin\theta\cos\tau - \cos\theta\sin\tau) \qquad (4.1.2)$$
$$L_Z = L\cos\tau\cos\phi$$

The magnitude of the lift force L can be related to the unit-less lift coefficient C_L by,

$$L = \sigma C_L v_{LG}^2 S/2 \qquad (4.1.3)$$

where

v_{LG} ≡ Aircraft Longitudinal Axis Velocity

σ ≡ *Air Density* (kg/m^3)

S ≡ Aircraft Wing Area (m^2)

Drag is the force along the longitudinal axis opposite the direction of travel. The drag force arises from resistance of the aircraft movement through the air. If DG is the magnitude of the drag, its components are:

$$DG_X = DG\cos\phi\cos\theta$$
$$DG_Y = DG\cos\phi\sin\theta \qquad (4.1.4)$$
$$DG_Z = -DG\sin\phi$$

The magnitude of the drag force D can be split into two components, the parasite drag C_{DO} and the induced drag C_{Di}. The parasite drag C_{DO} is due to the profile drag of the airfoil, skin friction drag, pressure drag, and interference drag of the other aircraft components. The induced drag is related by Prandtl's lifting line theory to the lift coefficient C_L by

(4.1.5) $$C_{Di} = C_L{}^2/(\Pi e a_r)$$

where

a_r ≡ Aircraft Wing Aspect Ratio

e ≡ Oswald Wing Efficiency Factor

The magnitude of the drag D is then related to these coefficients by,

(4.1.6) $$DG = \sigma v_{LG}{}^2 S(C_{D0} + C_L{}^2/\Pi e a_r)/2$$

The equations of motion can be found by equating the forces ((4.1.4), (4.1.1), (4.1.2)) for each of the x, y, z, components along with (4.1.3) and (4.1.6). This gives

(4.1.7)
$$\begin{aligned}
v_{LG}^{\bullet} &= g((T/W-(B + C_L C)v_{LG}{}^2) + \sin\phi) \\
\phi^{\bullet} &= g(C_L D v_{LG}\cos\tau - \cos\phi/v_{LG}) \\
\theta^{\bullet} &= -\, g C_L D v_{LG}\sin\tau/\cos\phi \\
x^{\bullet} &= v_{LG}\cos\phi\cos\theta \\
y^{\bullet} &= v_{LG}\cos\phi\sin\theta \\
z^{\bullet} &= -\, v_{LG}\sin\phi
\end{aligned}$$

where

$B = \sigma C_{DO} S/2W$

$C = \sigma S/2W\Pi e a_r$

$D = \sigma S/2W$

W ≡ Aircraft Weight (NT)

v_{LG} ≡ Aircraft Longitudinal Axis Velocity

The trajectory of the aircraft is determined by three controls; T (thrust), τ (bank angle), and C_L (coefficient of lift). All three controls are assumed to be instantaneously adjustable between the minimum and maximum values.

Aircraft dynamics are limited by a number of factors. Thrust (T) is limited to a maximum amount available (Thrust_Max). Maximum thrust is a characteristic of the design performance of the aircraft engine. We are also assuming fuel limitations. Once fuel is exhausted thrust is obviously no longer available. The fuel burn equation may be written as follows:

(4.1.8) $$F^{\bullet} = -e_f T$$

where

$F \equiv$ fuel amount

$e_f \equiv$ engine efficiency

$T \equiv$ time elapsed

A maximum amount of lift is also available at any velocity. The maximum lift available is related to the maximum value of C_L (C_L_Max). The maximum coefficient of lift (C_L_Max) is a parameter of design performance of the aircraft airframe. The maximum coefficient of lift (C_L_Max) determines the stall velocity of the aircraft. An uncontrollable loss of altitude occurs when an airplane is in a stall. The stall velocity is defined as the velocity (ν_{LG}) below the point at which the pitch rate ($\phi^{\bullet}$) defined in (4.1.7) can be held to zero, and is given by,

(4.1.9) $$\nu_{LG\text{–}Stall} = \left[\frac{\cos\phi}{C_L D \cos\tau} \right]^{1/2}$$

The maximum load factor of the airframe is another design parameter. The maximum load factor is given in G's which is equal to the force of gravity on an object at rest at sea level. The instantaneous number of G's for an aircraft is given by:

$$G = \frac{v_{LG}}{g} \left[(\theta^{\bullet}\cos\phi)^2 + (\phi^{\bullet})^2 \right]^{1/2}$$

where g is the gravitational acceleration constant.

4.1.2. NUMERICAL METHOD

We will now describe the numerical method for simulating the aerodynamic model discussed in Section 4.1.1. The first two equations of (4.1.54) can be decoupled from the rest and are of the vector form,

(4.1.10) $V^{\bullet} = F(V,U)$

where $F(\bullet,\bullet)$ is a $C^{(1)}$ function from R^{n+m} to R^n, V is a n-dimensional state vector, and U is a m-dimensional control vector.

The following notation will be needed. We define $t_i = t_{i-1} + h$, $V_i = V(t_i)$, $V^{\bullet}_i = V^{\bullet}(t_i)$, and $U_i = U(t_i)$. Denoting $F_V(\bullet,\bullet)$ and $F_U(\bullet,\bullet)$ as the matrix of partials with respect to the V and U coordinates of $F(\bullet,\bullet)$ respectively we define $F_V{}^i = F_V(V_i,U_i)$ and $F_U{}^i = F_U(V_i,U_i)$. We can now define the first order approximation to (4.1.10) at time t_{i+1} around t_i as,

(4.1.11) $V^{\bullet}_{i+1} = V^{\bullet}_i + F_V{}^i(V_{i+1} - V_i) + F_U{}^i(U_{i+1} - U_i)$

where we have used the fact that $V^{\bullet}_i = F(V_i,U_i)$. The trapezoidal rule allows us to approximate V_{i+1} in terms of V_i and is given by:

(4.1.12) $V_{i+1} = V_i + h(V^{\bullet}_{i+1} + V^{\bullet}_i)/2.$

Substituting (4.1.12) into (4.1.11) we get,

$$V^{\bullet}_{i+1} = V^{\bullet}_{i} + F_V^{i} V_i + hF_V^{i} V^{\bullet}_{i+1}/2 + hF_V^{i} V^{\bullet}_{i}/2 + F_U^{i}(U_{i+1} - U_i). \tag{4.1.13}$$

Collecting terms we have,

$$(I - hF_V^{i}/2)V^{\bullet}_{i+1} = (I + hF_V^{i}/2)V^{\bullet}_{i} + F_V^{i} V_i + F_U^{i}(U_{i+1} - U_i), \tag{4.1.14}$$

where I is the nxn identity matrix. Multiplying (4.1.14) by $(I - hF_V^{i}/2)^{-1}$ we have,

$$V^{\bullet}_{i+1} = (I - hF_V^{i}/2)^{-1}(I + hF_V^{i}/2)V^{\bullet}_{i} + (I - hF_V^{i}/2)^{-1}(F_V^{i} V_i + F_U^{i}(U_{i+1} - U_i)), \tag{4.1.15}$$

Equation (4.1.15) and equation (4.1.12) form the difference equation model for $\nu_{LG}^{\bullet}$ and $\phi^{\bullet}$ of (4.1.7), the aircraftdynamics differential model. We now give V_i, U_i, F_V^{i}, and F_V^{i} for the aircraft structural model.[29]

$$V_i = [\,\nu_{LGi} \quad \phi_i\,]^T \qquad U = [\,T_i \quad C_{Li} \quad \tau_i\,]^T$$

$$F_V^{i} = \begin{bmatrix} -2g(B + C_{Li})\nu_{LGi} & g\cos\phi_i \\ gC_{Li}D\cos\tau_i - g\cos\phi_i/(\nu_{LGi})^2 & g\sin\phi_i/\nu_{LGi} \end{bmatrix} \tag{4.1.16}$$

$$F_U^{i} = \begin{bmatrix} g/W & -gC(\nu_{LGi})^2 & 0 \\ 0 & gD\nu_{LGi}\cos\tau_i & -gC_{Li}D\nu_{LGi}\sin\tau_i \end{bmatrix}$$

---

[29] $[\;\;]^T$ denotes the transpose of the vector [ ]

We now turn our attention to the differential equation for $\theta^{\bullet}$ from (4.1.7). A close examination indicates that,

$$\theta^{\bullet}{}_{i+1}=\theta^{\bullet}(t_{i+1}) = -gC_{L(i+1)}D\nu_{LG(i+1)}\sin\tau_{i+1}/\cos\phi_{i+1} \tag{4.1.17}$$

is determined by V_{i+1} and U_{i+1} defined in (4.1.16) where V_{i+1} is determined by the methodology defined in (4.1.15) and (4.1.12) The trapezoidal rule is once again used along with (4.1.17) so that,

$$\theta_{i+1} = \theta(t_{i+1}) = \theta_{i+1} + h(\theta^{\bullet}{}_{i+1} + \theta^{\bullet}{}_{i})/2 \tag{4.1.18}$$

The last equations for $x^{\bullet}$, $y^{\bullet}$, and $z^{\bullet}$ of (4.1.7) are decoupled and can be determined in terms of V_{i+1}, U_{i+1} and θ_{i+1} and are given by

$$\begin{aligned} x^{\bullet}{}_{i+1} &= \nu_{LG(i+1)}\cos\phi_{i+1}\cos\theta_{i+1} \\ y^{\bullet}{}_{i+1} &= \nu_{LG(i+1)}\cos\phi_{i+1}\sin\theta_{i+1} \\ z^{\bullet}{}_{i+1} &= -\nu_{LG(i+1)}\sin\phi_{i+1} \end{aligned} \tag{4.1.19}$$

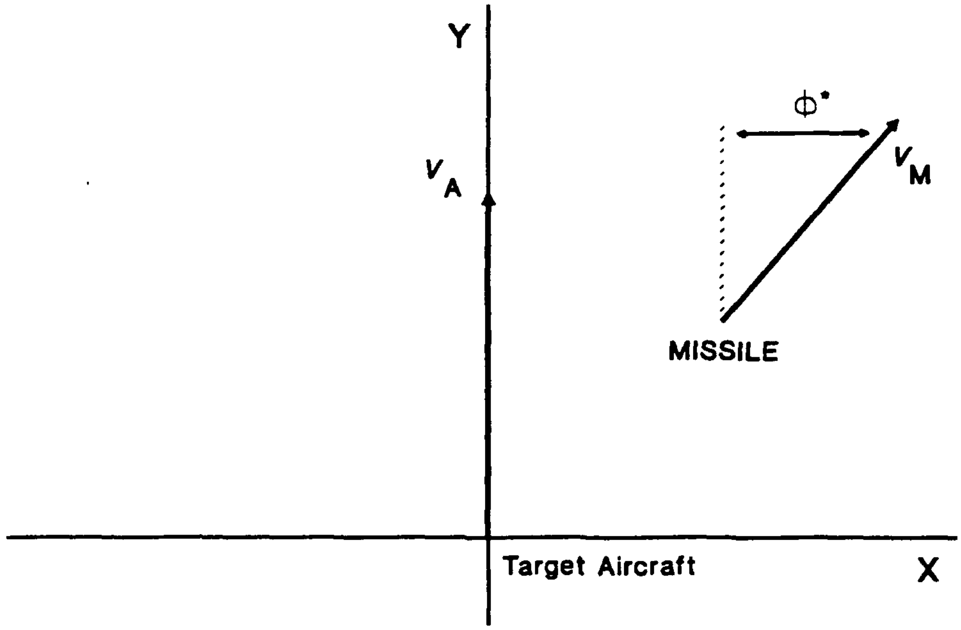
We use the trapezoidal rule one final time along with (4.1.19) to get:

$$\begin{aligned} x_{i+1} &= x_i + h(x^{\bullet}{}_{i+1} + x^{\bullet}{}_{i})/2 \\ y_{i+1} &= y_i + h(y^{\bullet}{}_{i+1} + y^{\bullet}{}_{i})/2 \\ z_{i+1} &= z_i + h(z^{\bullet}{}_{i+1} + z^{\bullet}{}_{i})/2 \end{aligned} \tag{4.1.20}$$

4.1.3 PROBABILISTIC MISSILE MODEL DEVELOPMENT

We will now derive the probabilistic model for missile firing envelopes. This model will serve as the causative attrition rules G_i between aircraft. The true nature of these models are classified and we only seek a plausible approximation. The actual details of the model is not critical, but the methodology to determine the firing and risk surfaces is critical to our methodology.

We start by asymptotically looking at the deterministic case for a constant speed missile with bounded turn rate against a straight flying aircraft with constant speed taking place at fixed altititude. Justification for this asymptotic case is that the vast majority of pilots shot down never realized they had been fired on and hence took no evasive maneuvers. The missile is assumed to have *a finite life time* which is the burn time of it motor. Our problem is then to determine the maximum range from the target aircraft that the missile can reach. The coordinate frame we will work in is centered at the target aircraft with the aircraft's velocity vector aligned with the y-axis. This coordinate system is shown in Figure 4.1.1.



COORDINATE SYSTEM RELATIVE TO TARGET AIRCRAFT

FIGURE 4.1.1

The analysis is started by assuming the missile is infinitely maneuverable. This corresponds to the maximum firing range from any point in space since the missile can always be fired toward the optimal intercept point regardless of the position and direction of the aircraft firing the missile.

The equations of motion are:

$$X^{\bullet} = V_M \sin\Phi$$
$$Y^{\bullet} = -V_A + V_M \cos\Phi \qquad \textbf{(4.1.21)}$$

where

V_M = velocity of missile
V_A = velocity of target aircraft
Φ = missile heading control.

The Hamiltonian of the optimal minimum intercept time control problem is:

$$H = 1 + \Gamma_X V_M \sin\Phi + \Gamma_Y V_M \cos\Phi - \Gamma_Y V_A \qquad \textbf{(4.1.22)}$$

The adjoint equations are then:

$$\Gamma_X^{\bullet} = 0$$
(4.1.23)
$$\Gamma_Y^{\bullet} = 0$$

The optimai control found by minimizing the Hamiltonian with respect to Φ is, $\Phi = \arctan(\Gamma_Y/\Gamma_X) + \pi$. The fact that the adjoints are constant implies that Φ is constant and may be found geometrically by solving for:

$$X(t_f) = X(t_0) + t_f V_M \sin\Phi = 0$$
(4.1.24)
$$Y(t_f) = Y(t_0) - t_f V_A + t_f V_M \cos\Phi = 0.$$

The heading angle Φ is then,

(4.1.25) $$\Phi = \arctan(X(t_0), Y(t_0) - t_f V_A) + \pi$$

Our interest lies in determining the manifold of the maximum range of the missile for t_f = missile burn time. This leads to the equation:

(4.1.26) $$X^2 + (Y - V_A T_{BT})^2 = (V_M T_{BT})^2.$$

The maximum range for a missile fired optimally at the intercept point is thus an eccentric circle in front of the target aircraft by the distance the target can fly during the life of the missile with radius of the distance the missile can travel during its life. An example is shown in Figure 4.1.2 below.

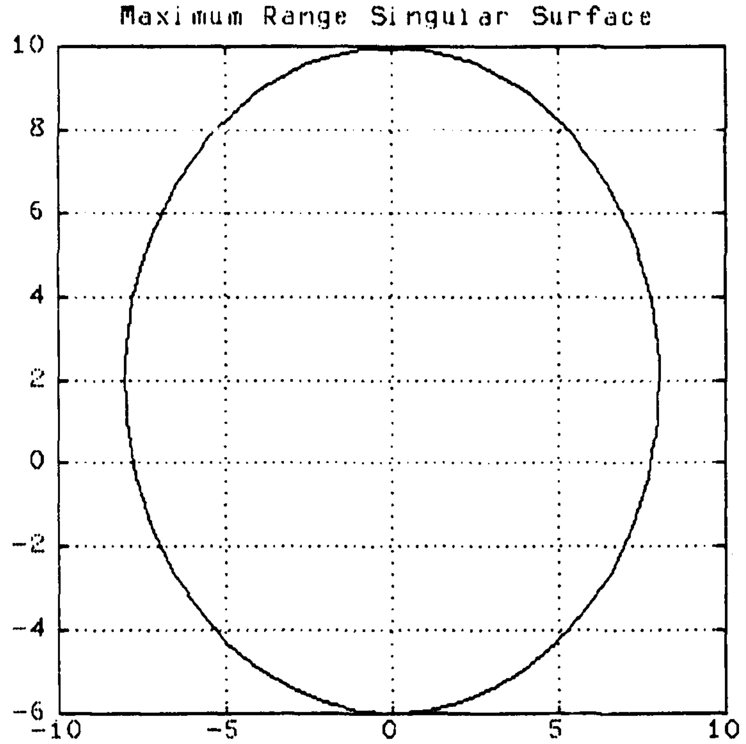


FIGURE 4.1.2

We now may consider the case when the missile has a maximum turn rate. The case above serves as a singular surface for when a missile is perfectly aligned for intercept when fired. If the missile is not aligned then the maximum range will be reduced as the missile wastes time and energy turning to an intercept course.

The equations of motion are now:

$$(4.1.27)\qquad \begin{aligned} X^{\bullet} &= V_M \sin\Phi \\ Y^{\bullet} &= -V_A + V_M \cos\Phi \\ \Phi^{\bullet} &= V_M \phi / R \end{aligned}$$

where

V_M = velocity of missile
V_A = velocity of target aircraft
Φ = missile heading relative to aircraft heading
ϕ = missile turn control ($-1 \leq \phi \leq 1$)
R = minimum turn radius of missile.

It should be noted that Φ is no longer a control, but now a state variable. The Hamiltonian of the optimal minimum intercept time control problem is:

(4.1.28) $$H = 1 + \Gamma_X V_M \sin\Phi + \Gamma_Y V_M \cos\Phi - \Gamma_Y V_A + \Gamma_\Phi V_M \phi / R$$

The adjoint equations become:

(4.1.29) $$\begin{aligned} \Gamma_X^{\bullet} &= 0 \\ \Gamma_Y^{\bullet} &= 0 \\ \Gamma_\Phi^{\bullet} &= -V_M(\Gamma_X \cos\Phi - \Gamma_Y V_M \sin\Phi) \end{aligned}$$

The optimal control ϕ is found by minimizing the Hamiltonian with respect to ϕ yielding:

(4.1.30) $$\phi = -\mathrm{sign}(\Gamma_\Phi)$$

The terminal conditions are :

(4.1.31) $$X(t_f) = Y(t_f) = 0; \;\; \Phi(t_f) = \pi + \theta$$ [30].

Transversality also yields $\Gamma_\Phi(t_f) = 0$. The terminal conditions for the other adjoints follow from the terminal condition that $\min_\phi H = 0$ yielding,

(4.1.32) $$\Gamma_X = \frac{\sin\theta}{V_M + V_A \cos\theta} \qquad \Gamma_Y = \frac{\cos\theta}{V_M + V_A \cos\theta} .$$

---

[30] θ can be viewed as the angle at which intercept is approached infinitesimally

This leads to

$$\textbf{(4.1.33)} \qquad \Gamma_{\Phi}^{\bullet}(t_f) = \frac{-V_M}{V_M + V_A\cos\theta}\,(\sin\theta\cos\Phi(t_f) - \cos\theta\sin\Phi(t_f))$$

$$= \frac{-V_M}{V_M + V_A\cos\theta}\,\sin(\theta - \Phi(t_f))$$

$$= 0.$$

This implies that $\Phi^{\bullet}(t_f) = 0$. The conclusion is that all trajectories terminate on a singular surface on which the missile is flying a straight intercept course. The missile thus turns maximally until it reaches a point that $\Phi(t)$ matches the angle for $X(t)$ and $Y(t)$ that would be optimal for the infinitely maneuverable missile. The missile then switches to a straight lead intercept trajectory. A typical trajectory shape is shown in Figure 4.1.3 below.

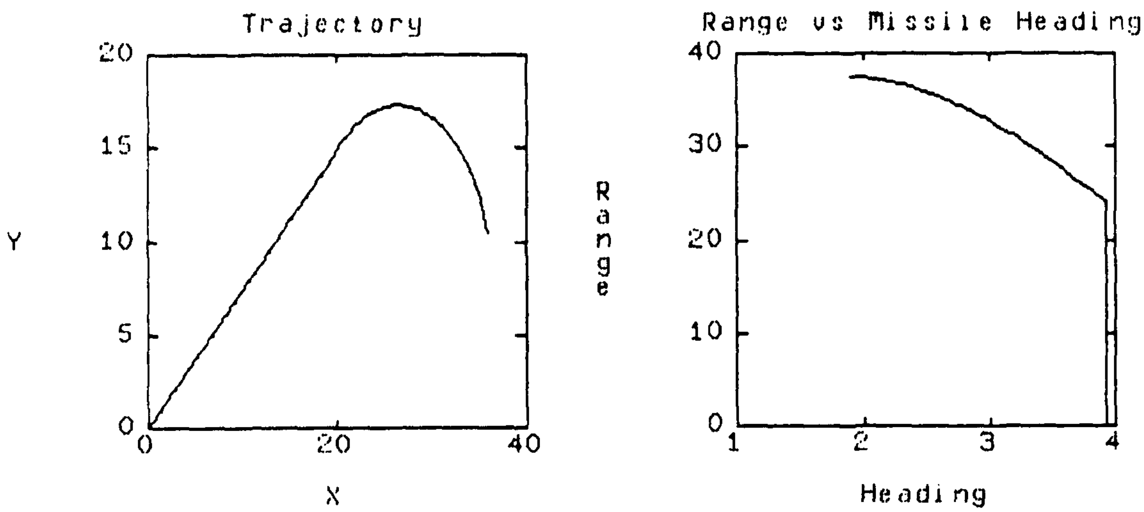


FIGURE 4.1.3

Letting τ be retro time, and τ_1 be the retro time at which the trajectory enters the singular surface we have the equations of motion for $\tau \le \tau_1$,

$$\Phi(\tau) = \pi + \theta$$

$$\text{(4.1.34)} \qquad X(\tau) = \tau V_M \sin\theta$$

$$Y(\tau) = \tau(V_A + V_M\cos\theta)$$

The equations for motion for $\tau > \tau_1$ when $\phi = 1$ are:

$$\Phi(\tau) = \pi + \theta - \frac{V_M}{R}(\tau - \tau_1)$$

$$\text{(4.1.35)} \qquad X(\tau) = R\cos\left[\theta - \frac{V_M}{R}(\tau-\tau_1)\right] + X(\tau_1) - R\cos\theta$$

$$Y(\tau) = -R\sin\left[\theta - \frac{V_M}{R}(\tau-\tau_1)\right] + V_A(\tau-\tau_1) + Y(\tau_1) + R\sin\theta$$

Our interest lies in how the maximum range envelope varies with deviation from the optimal missile intercept heading for the infinitely maneuverable case. Numerically we can solve for the maximum range as a function of heading error from the Maximum Singular Surface Range for the infinitely maneuverable missile solved earlier. The outermost range envelope corresponds to the case where the missile heading exactly matches the firing heading if the missile was infinitely maneuverable. Thus for each X and Y there corresponds the optimum firing angle $\Phi^*(X,Y)$ which is the solution to (4.1.25). Then for each angle Ω = arctan(Y,X) we can determine numerically the maximum X, Y pair for a fixed error offset from $\Phi^*(Y\tan\Omega,Y)$. These plots are given in Figure 4.1.4.

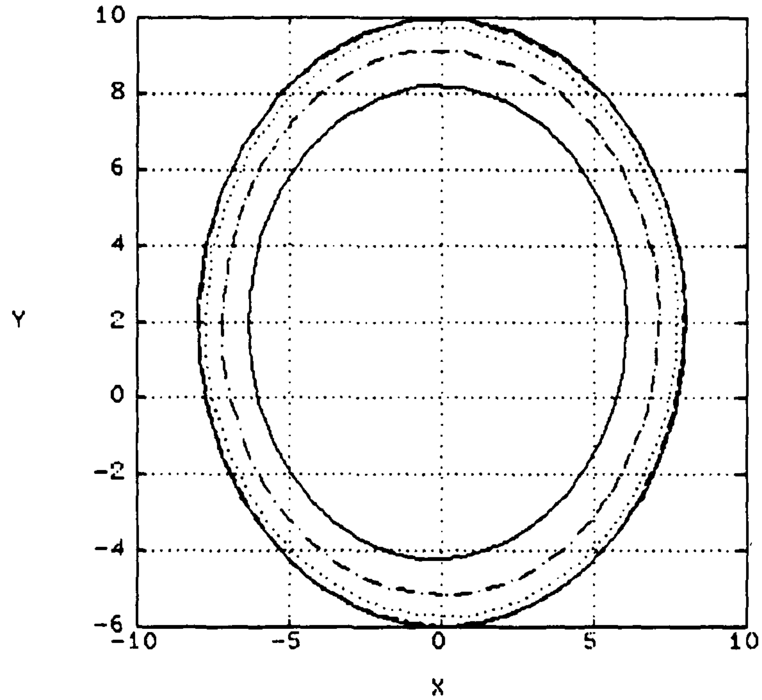
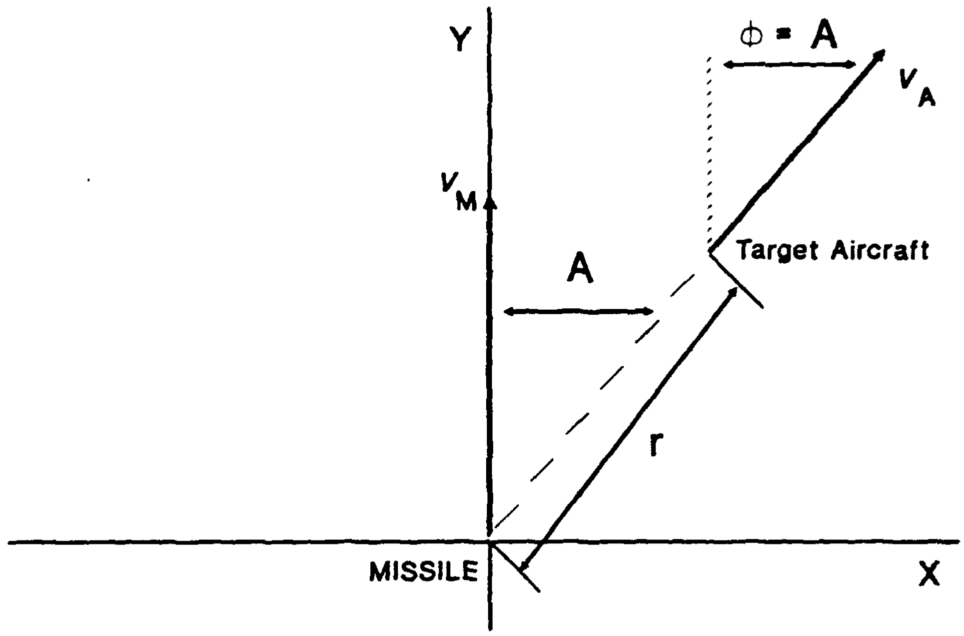


FIGURE 4.1.4
FIRING RANGES

The next asymptotic case will now be considered. We consider an infinitely maneuverable aircraft against a missile with bounded turn radius. Once again we assume the encounter takes place at fixed altitude with constant velocity for both vehicles. Since the aircraft is now infinitely maneuverable we assume the aircraft always aligns its velocity vector outward along the line of sight between the aircraft and missile. The coordinate frame we will now work in is centered at the missile with the Y-axis aligned with the missiles velocity vector. The coordinate system is shown in Figure 4.1.5.



COORDINATE SYSTEM RELATIVE TO MISSILE

FIGURE 4.1.5

The equations of motion written in polar coordinates are:

$$r^{\bullet} = V_A - V_M \cos A \tag{4.1.36}$$

$$A^{\bullet} = V_M \left[\frac{\phi}{R} \quad \frac{\sin A}{r} \right]$$

where

r = distance of aircraft from missile
A = heading of aircraft relative to missile heading
V_M = velocity of missile
V_A = velocity of target aircraft
ϕ = missile turn control ($-1 \leq \phi \leq 1$)
R = minimum turn radius of missile.

The Hamiltonian is:

$$H = \left[1 + V_A \Gamma_r + \Gamma_A \frac{V_M \phi}{R} \right] + \Gamma_A \frac{V_M \sin A}{r} - \Gamma_r V_M \cos A \tag{4.1.37}$$

The adjoint equations are then

$$(4.1.38)\qquad \begin{aligned} \Gamma_r^{\bullet} &= -\Gamma_A \frac{V_M \sin A}{r^2} \\ \Gamma_A^{\bullet} &= -V_M \left[\Gamma_A \frac{\cos A}{r} + \Gamma_r \sin A \right] \end{aligned}$$

Minimizing H with respect to ϕ implies the optimal control:

$$(4.1.39)\qquad \phi = -\text{sign}[\Gamma_A]$$

The terminal conditions require some explanation. It should be obvious that the only way $r = 0$ can be reached is along the singular surface for which $A = 0$. Thus we seek those trajectories that reach the singular surface $A=0$, $r = s$ in minimum time. Symmetry arguments clearly indicate that $\phi = 1$ in the right half plane and $\phi = -1$ in the left half plane. Assuming the missile has a reasonably long burn time, we can use for the boundary of the range of the missile:

$$(4.1.40)\qquad A^{\bullet} = V_M \frac{\phi}{R}$$

Letting τ once again be retrograde time, and τ_1 be the retrograde time when the breakaway from $A = 0$ occurs, we have for the right half plane ($\phi = 1$) for $\tau > \tau_1$:

$$(4.1.41)\qquad \begin{aligned} A(\tau) &= \frac{V_M}{R} (\tau - \tau_1) \\ r(\tau) &= R\sin\left[\frac{V_M}{R} (\tau - \tau_1) \right] + V_A(\tau - \tau_1) + \tau_1(V_M - V_A) \end{aligned}$$

To find the maximum capture range envelope we first note that we can relate r to A by,

$$(4.1.42)\qquad r = R\sin A + (V_M - V_A)T_{BT} - R \bullet A$$

where T_{BT} is the life time of the missile engine burn. The two figures below show how the maximum range envelope varies with missile burn time and minimum turn radius. The first figure shows how the maximum range envelope varies for fixed minimum turn radius R, while T_{BT} is varied. The second figure indicates how the maximum range envelope varies as R is varied while T_{BT} is held fixed.

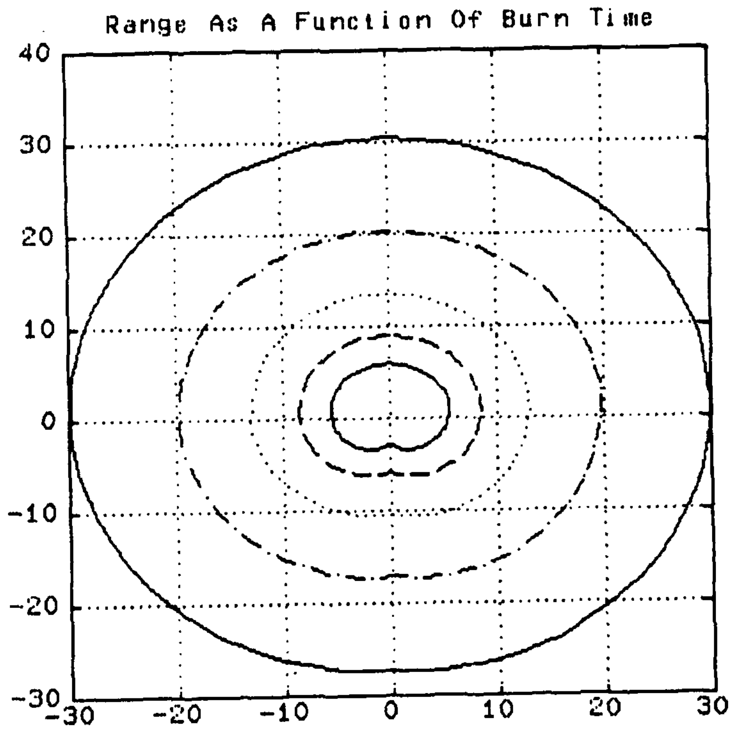


FIGURE 4.1.6

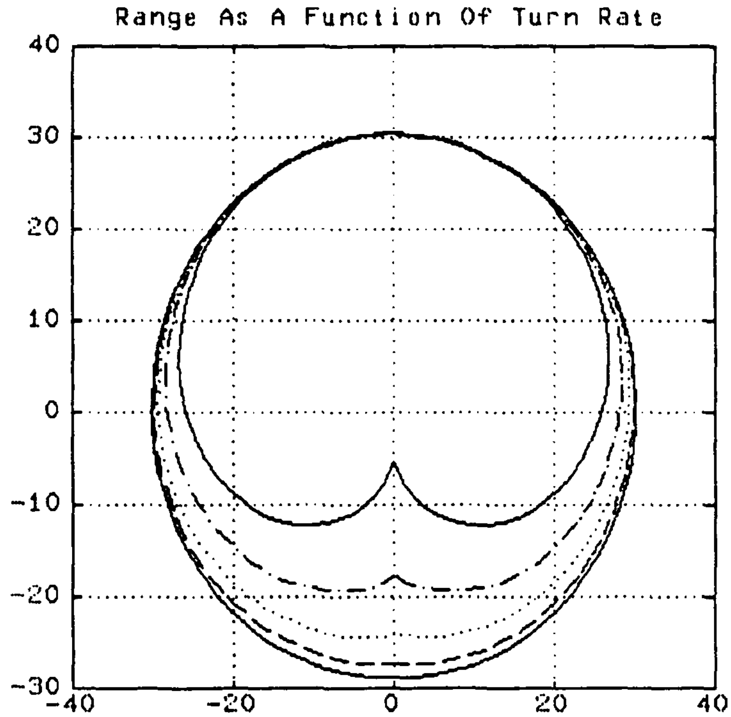


FIGURE 4.1.7

We now will splice our deterministic asymptotic results together into a probabilistic missile effectiveness model. Arguably the results are not rigorous. We seek only a plausible model of sufficient complexity that our methodology can be exercised in more than a trivial fashion.

Earlier we mentioned that the vast majority of pilots shot down never realized they had been fired upon. Therefore we define P_U = probability that a missile launch is undetected by the fired upon aircraft. We can thus use the deterministic model summarized by equation (4.1.26). Rewriting equation (4.1.26) in line of sight coordinates:

(4.1.43)
$$r = V_A T_{BT} \cos(A_{ASP}) + \left[(V_M T_{BT})^2 - (V_A T_{BT} \sin(A_{ASP}))^2 \right]^{1/2}$$

where A_{ASP} is the aspect angle which is defined as the angle measured clockwise between the aircraft heading and the line of sight, and r is the line of sight distance.

The next step is to modify the model given by equation 4.1.43 from a deterministic model to a probabilistic one. This modification will account for the fact that all of the parameters T_{BT}, V_M, V_A, etc. vary probabilistically. The probability that the missile will destroy an aircraft from a line of sight distance r, where the aircraft has aspect angle A_{ASP}, is given by:

(4.1.44) $$P(K \mid r, A_{ASP}, U) = \begin{cases} \beta\exp(-\alpha d), \text{ if } d \geq 0 \\ \beta, \text{ otherwise} \end{cases}$$

where

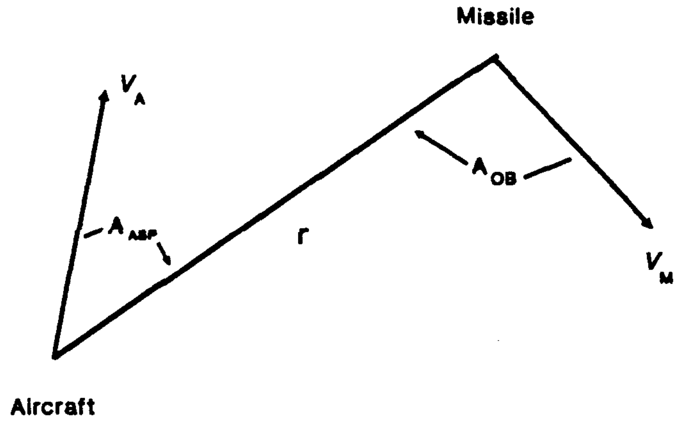
$$d = r - V_A T_{BT} \cos(A_{ASP}) - \left[(V_M T_{BT})^2 - (V_A T_{BT} \sin(A_{ASP}))^2 \right]^{1/2}.$$

The non-negative α models the variability around the nominal parameters V_A, V_M, and T_{BT}. Last ditch end-game maneuvers, electronic counter measures, terrain masking, etc are modeled by the parameter $0 < \beta < 1$.

Equation (4.1.26) was the result when the missile was assumed to be infinitely maneuverable so that missile off-boresight angle was not considered. The off-boresight angle firing error from equation (4.1.25) can be written as:

(4.1.45) $$\Phi_e = A_{ASP} - A_{OB} + \pi + \arctan(r\sin(A_{ASP}), r\cos(A_{ASP}) - T_{BT} V_A)$$

where Φ_e is the off-boresight angle error and A_{OB} is the off-boresight angle measured clockwise between the missile heading and the line of sight. The line of sight coordinate system is shown in Figure (4.1.8).



Line Of Sight Coordinates

FIGURE 4.1.8

The effect of A_{OBe} was shown in Figure 4.1.4. As it turns out, the relationship between the maximum missile range and A_{OBe} is highly nonlinear and a closed form solution does not exist. The radii of the eccentric circles are found from Equations (4.1.35) to be:

$$\mu^2 = X^2 + (Y - V_A)^2 \tag{4.1.46}$$

$$= \left[V_M T_{BT} - V_M (T_{BT} - \tau_1) \right]^2$$

$$+ 2 R_M^2 \left[1 - \cos \left[\frac{V_M}{R_M} (T_{BT} - \tau_1) \right] \right]$$

$$+ 2 \tau_1 V_M R \sin \left[\frac{V_M}{R_M} (T_{BT} - \tau_1) \right]$$

where the parameters are those of Equations (4.1.27).[31] Define θ' by

$$\theta' = \arctan(X(t_0), Y(t_0) - t_f V_A), \tag{4.1.47}$$

---

[31] T_{BT} is still the missile burn time.

which is the angle between the initial position of the missile and the heading of the aircraft. Φ_e can then be expressed as:

$$\textbf{(4.1.48)} \qquad \Phi_e = \theta - \theta' - \frac{V_M}{R_M}\,(T_{BT} - \tau_1).$$

The parameter C is defined below, strictly for convenience as:

$$\textbf{(4.1.49)} \qquad C = -\frac{V_M}{R_M}\,(T_{BT} - \tau_1)$$

The problem is to determine the unknowns θ and τ_1 from Φ_e so that μ may be calculated. Again, no closed form solution exists. Our methodology is to use a second order Taylor expansion.

Assuming Φ_e is negative, the first step is the equation found by combining second order Taylor expansions of equations (4.1.35) with (4.1.47):

$$\textbf{(4.1.50)} \qquad \tan\theta' = \frac{X}{Y - V_A T_{BT}} \approx \frac{2V_M T_{BT}\sin\theta - R_M(C)^2\cos\theta}{2V_M T_{BT}\cos\theta - R_M(C)^2\sin\theta}$$

$$= \tan\left[\theta - \tan^{-1}\left[\frac{R_M(C)^2}{2V_M T_{BT}}\right]\right]$$

This leads us to conclude that:

$$\textbf{(4.1.51)} \qquad \theta' \approx \theta - \tan^{-1}\left[\frac{R_M(C)^2}{2V_M T_{BT}}\right].$$

A second order Taylor expansion can now be applied to equation (4.1.51), yielding

$$\theta \approx \theta' + \frac{R_M}{2}\left[\frac{C^2}{V_M T_{BT}}\right]$$

The results of equations (4.1.51) and (4.1.48), valid for negative Φ_e, can be extended to determine the radius of the eccentric circle missile range μ approximately, as a function of $|\Phi_e|$, from equation (4.1.46) as:

$$\begin{aligned}
\mu^2 \approx{}& (V_M T_{BT})^2 - 2R_M|\Phi_e|\, V_M T_{BT} + \\
& 2R_M{}^2\left[1-\cos\left[\frac{V_M T_{BT}}{R_M} - \left[\frac{V_M T_{BT}}{R_M}\left[\frac{V_M T_{BT}}{R_M} - 2|\Phi_e|\right]\right]^{1/2}\right]\right] \\
& + \left[\frac{V_M T_{BT}}{R_M}\left[\frac{V_M T_{BT}}{R_M} - 2|\Phi_e|\right]\right]^{1/2} * \\
& \sin\left[\frac{V_M T_{BT}}{R_M} - \left[\frac{V_M T_{BT}}{R_M}\left[\frac{V_M T_{BT}}{R_M} - 2|\Phi_e|\right]\right]^{1/2}\right].
\end{aligned} \tag{4.1.52}$$

The probabilistic model of Equation (4.1.44) can now be extended. The probability that a missile destroys an aircraft from a line of sight distance r, with aircraft aspect angle A_{ASP}, and off boresight angle A_{OB} when a launch is undetected is given by:

$$P(K \mid r, A_{ASP}, A_{OB}, U) = \begin{cases} \beta\exp(-\alpha d'), & \text{if } d' \geq 0 \\ \beta, & \text{otherwise} \end{cases}$$

where

$$\Phi_e = A_{ASP} - A_{OB} - \arctan(r\sin(A_{ASP}), r\cos(A_{ASP}) - T_{BT}V_A)$$

and

$$d' = r - V_A T_{BT}\cos(A_{ASP})$$

$$- \left[(V_M T_{BT})^2 - V_A T_{BT}\sin(|A_{ASP}|) - 2R_M|\Phi_e| V_M T_{BT} \right.$$

$$+ 2R_M{}^2\left[1 - \cos\left[\frac{V_M T_{BT}}{R_M} - \left[\frac{V_M T_{BT}}{R_M}\left[\frac{V_M T_{BT}}{R_M} 2|\Phi_e| \right]\right]^{\frac{1}{2}} \right]\right]$$

$$+ 2R_M{}^2\left[\frac{V_M T_{BT}}{R_M}\left[\frac{V_M T_{BT}}{R_M} - 2|\Phi_e| \right]\right]^{\frac{1}{2}} \quad *$$

$$\left. \sin\left[\frac{V_M T_{BT}}{R_M} - \left[\frac{V_M T_{BT}}{R_M}\left[\frac{V_M T_{BT}}{R_M} - 2|\Phi_e| \right]\right]^{\frac{1}{2}} \right]\right]^{\frac{1}{2}}$$

The case when a missile launch is detected by the target aircraft will now be considered. The probability that a launch is not detected is given by $P_D = 1 - P_U$. The assumption for a detected missile launch is that the target aircraft tries to outrun the missile. The deterministic version for an infinitely maneuverable aircraft was given by Equation (4.1.42), which (rewritten in line of sight coordinates) is given by:

(4.1.54) $$r = R_M\sin(A_{OB}) + (V_M - V_A)T_{BT} - R_M A_{OB}.$$

Equation (4.1.54) assumed that the target aircraft could instantaneously align its velocity vector outward along the line of sight. Now an approximation will modify this equation to account for the energy and time needed for this alignment by the target aircraft. The equations of motion in line of sight coordinates are:

$$r^{\bullet} = -\left[V_A \cos(A_{ASP}) + V_M \cos(A_{OB}) \right]$$

$$\text{(4.1.55)} \qquad A_{OB}^{\bullet} = \frac{1}{r}\left[V_A \sin(A_{ASP}) + V_M \sin(A_{OB}) \right] - \frac{V_A}{R_A}\,\phi$$

$$A_{ASP}^{\bullet} = \frac{1}{r}\left[V_A \sin(A_{ASP}) + V_M \sin(A_{OB}) \right] - \frac{V_M}{R_M}\,\theta,$$

where we have introduced R_A for the turn radius of the aircraft, with $\phi \in [-1,1]$ being the missile control, while the aircraft control $\theta \in [-1,1]$. An approximation to the loss in distance along the line of sight of the aircraft due to its turning outward ($A_{ASP} = \pi$) is given by $R_A(\sin(|A_{ASP}|) - \pi + |A_{ASP}|)$.[32]

Similarly to the undetected missile case we combine the results into a probabilistic model as[33]:

$$\text{(4.1.56)} \qquad P(K\,|\,r, A_{ASP}, A_{OB}, D) = \begin{cases} \beta\exp(-\alpha d''), & \text{if } d'' \geq 0 \\ \beta, & \text{otherwise} \end{cases}$$

where

$$d'' = r - R_M(\sin(|A_{OB}|) - |A_{OB}|) - (V_M - V_A)T_{BT} - R_A(\pi - |A_{ASP}| - \sin(|A_{ASP}|)).$$

---

[32]Note we have assumed the missile is launched from a distance such that $r^{-1} << V_A/R_A$.

[33]Note α and β have the same meaning as in the undetected case.

Equations (4.1.53) and (4.1.55) give a probabilistic missile effectiveness model in terms of nominal parameters. This model does not directly take altitude into account as in general this is a fast state variable that has little effect as far as initial altitude separation at launch. The maximum turn rates and velocities are effected by altitude and airframe load factors. This remain near a nominal level for a launch at given altitude. Missiles such as AMRAAM normally seek a nominal mid-flight altitude to optimally set these parameters. This model display a reasonable approximation to actual missile firings sufficient for our purposes.

4.1.4. AIRCRAFT FRAME DESCRIPTION

The aircraft frame and relational subframes can now be described. The details of the attached procedures are found in other sections such as 4.1.3, 4.1.2, and 6.

Aircraft *Frame*

- Mesarovic Slot
  - Parameters:
    - Max Clift: a real
    - Max Thrust: a real
    - Max G's: a real
    - Weight: a real
    - Wing Surface Area: a real
    - Oswald Efficiency: a real
    - Wing Aspect Ratio: a real
  - State:
    - X: a real
    - Y: a real
    - Z: a real
    - V_{LG}: a real
    - A_{LG}: a real
    - Heading Rate: a real
    - Pitch Rate: a real
    - *Heading:* [-Pi,Pi]
    - Pitch: (-Pi/2,Pi/2)
    - Fuel: a real
    - Number_Air_Air_Missiles: [0,1,2,3,4,5,6]
    - Number_Air_Ground_Missi: [0,1,2,3,4,5,6]
  - Controls:
    - Bank: (-Pi/2,Pi/2)
    - Thrust: (0, Max_Thrust)
    - Clift: (0, Max_Clift)
  - *if-needed* (procedure to update state)
- Relational State Slot
  - Tactic: (LOS_Pursuit, LOS_Evade, LOS_Ground_Pursuit, LOS_Ground_Evade Harrier_Attack_F16 Harrier_Evade_f16 F16_Evade_Harrier F16_Attack_Harrier)
- Causal Attrition Slot
  - Air To Air Missile Type Attrition:
    - Missile Description:
      - Nominal Burn TIME: a real
      - Nominal Velocity: a real
      - $\alpha_{Detected}$: a positive real
      - $\beta_{Detected}$: [0,1]
      - $\alpha_{Undetected}$: a positive real

- $\beta_{Undetected}$: [0,1]
- Prob Launch Det: [0,1]
- *if-needed* (procedure to determine if target destroyed)
- Air To Ground Missile Type Attrition:
  - Missile Description:
    - Nominal Burn TIME: a real
    - Nominal Velocity: a real
    - α: a positive real
    - β: [0,1]
  - *if-needed* (procedure to determine if target destroyed)
- Self Attrition Slot
  - if-needed (procedure to test for crash)

Harrier Attack of F16 *Frame*

- Harrier Description:
  - Speed: a real
  - Kill Radii: a real
  - Kill Eccentricity: a real
- F16 Type Target: an aircraft Frame
- F16 Description:
  - Speed: a real
  - Kill Radii: a real
  - Kill Eccentricity: a real
  - Max Turn Radius: a real

if-needed (procedure to determine Harrier control)

Harrier Evade F16 Frame

- Harrier Description:
  - Speed: a real
  - Kill Radii: a real
  - Kill Eccentricity: a real
- F16 Type Target: an aircraft Frame
- F16 Description:
  - Speed: a real
  - Kill Radii: a real
  - Kill Eccentricity: a real
  - Max Turn Radius: a real

if-needed (procedure to determine Harrier control)

F16 Evade Harrier Frame

- Harrier Type Target: an aircraft Frame
- Harrier Description:
  - Speed: a real
  - Kill Radii: a real
  - Kill Eccentricity: a real
- F16 Description:
  - Speed: a real

Kill Radii:	a real
Kill Eccentricity:	a real
Max Turn Radius:	a real

if-needed (procedure to determine F16 control)

F6 Attack Harrier Frame

Harrier Type Target:	an aircraft Frame
Harrier Description:	
Speed:	a real
Kill Radii:	a real
Kill Eccentricity:	a real
F16 Description:	
Speed:	a real
Kill Radii:	a real
Kill Eccentricity:	a real
Max Turn Radius:	a real
Advasary:	Aircraft Frame

if-needed (procedure to determine F16 control)

LOS Pursuit *Frame*

Advasary:	Aircraft Frame

if-needed (procedure to determine LOS heading control)

LOS Evade *Frame*

Advasary:	Aircraft Frame

if-needed (procedure to determine LOS heading control)

LOS Ground Pursuit *Frame*

Fixed Altitude:	a positive real
Advasary:	SAM Frame, Ground Target Frame

if-needed (procedure to determine LOS heading control)

LOS Ground Evade *Frame*

Fixed Altitude:	a positive real
Advasary:	SAM Frame, Ground Target Frame

if-needed (procedure to determine LOS heading)

4.2. SAMS (Surface To Air Missile Sites)

The next component type to be discussed is SAM sites. The probabilistic model for the missile capabilities for simplicity is assumed to have the same form as in section 4.1.3. The difference between air to air, and air to ground missiles will be imbedded in nominal parameters such as V_M, T_{BT}, P_D, etc. The location of the SAM sites is fixed but may be unknown to other components.

SAM *Frame*
- Mesarovic Slot
  - Parameters:
    - X: a real
    - Y: a real
  - State:
    - Number_Ground_Air_Missl: [0,1,2,3,4,5,6]
- Causal Attrition Slot
  - Ground To Air Missile Type Attrition:
    - Missile Description:
      - Nominal Burn TIME:a real
      - Nominal Velocity: a real
      - $\alpha_{Detected}$: a positive real
      - $\beta_{Detected}$: [0,1]
      - $\alpha_{Undetected}$: a positive real
      - $\beta_{Undetected}$: [0,1]
      - Prob Launch Det: [0,1]
    - *if-needed* (procedure to determine if target destroyed)

4.3. HOMES (Home Aircraft Bases)

The Home component type describes airfields. Home components have fixed location, and their purpose is to reproduce (takeoff), or cause attrition (landing) of aircraft components. The frame is shown below.

HOME *Frame*
- Mesarovic Slot
  - Parameters:
    - X: a real
    - Y: a real
  - State:
    - Number_Aircraft_Type_A: a pos integer
    - Number_Aircraft_Type_B: a pos integer
- Causal Attrition Slot
  - *if-needed* (procedure to land aircraft)
- Reproductive Slot
  - *if-needed* (procedure to takeoff aircraft)

4.4. GROUND TARGETS

The last component type to be considered are Ground Targets. Components of this type are targets of tactical interest. Ground target real world counterparts could be factories, or other facilities of importance. This very simple frame is shown below.

Ground Target *Frame*
Mesarovic Slot
Parameters:

X:	a real
Y:	a real

5. SYSTEM INTEGRATION

This section outlines how to integrate all the concepts discussed throughout this dissertation together via the semantic control paradigm. The use of the probabilistic missile models will be discussed, and how to integrate the results into determining and using the appropriate differential game model. Methodologies for implementing the system goal selector will also be explored and outlined. The goal selector determines the differential game models, and from the knowledge base. Three possible methodologies for the goal selector will be discussed. Lastly the control adapter will be discussed. The identifier is not discussed as perfect information is assumed.

5.1. FIRING AND AVOIDANCE SURFACES AS GOALS

Two surfaces must be determined, a firing surface, and an avoidance surface. The firing surface is a function of resource allocation and a desired probability of being able to destroy all assigned targets for a mission. A simplified equation for the probability of being able to destroy all assigned aircraft targets (P_M), based on the negative binomial distribution is given by:

$$P_M = \sum_{x=K}^{N} \frac{(x-1)!}{(x-K)!(K-1)!} p^K (1-p)^{X-K} . \tag{5.1}$$

where

N ≡ Number Of Missiles Remaining On Aircraft
p ≡ Probability That A Missile Will Destroy Its Target
K ≡ Number Of Targets
x ≡ Number Of Missiles Fired.

Equation (5.1) is solved for p given P_M, N and K. A more complex equation might *include a tradeoff on the cost* of a missile, or scarcity of the missiles as a resource. Suppose that an aircraft has 6 air to air missiles, and the expected number of encounters is 2 aircraft targets. To assure a 90% probability of being able to destroy the expected targets, each missile must be fired such that the missile has a 58% probability of destroying its target.

The next step is to use equations (4.1.53) and (4.1.56) to determine the firing surface. Figures 5.1 and 5.2 show the firing surface for a missile to have a 58% probability of destroying its target when Vm = 1, Rm = 1, Va = .25, Ra = 4, Tbt = 9, $\alpha = 0.25$, $\beta = 0.9$, and $P_D = 0.2$.

FIGURE 5.1

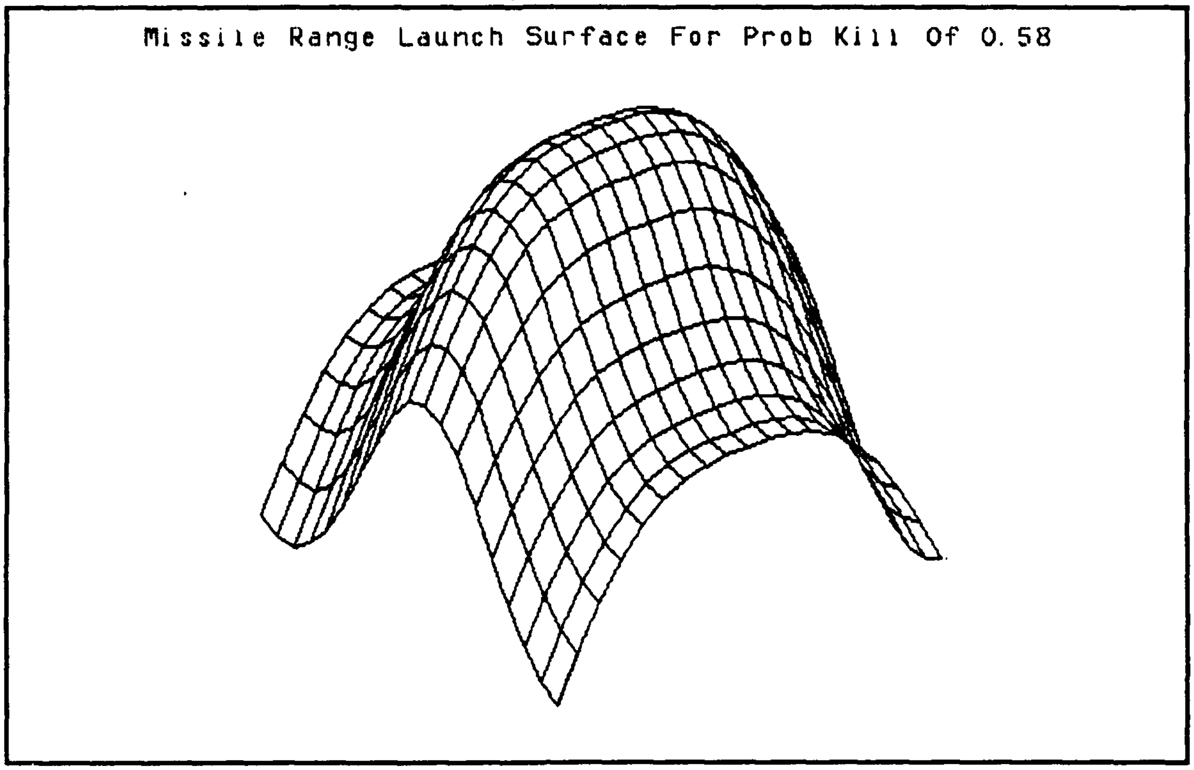
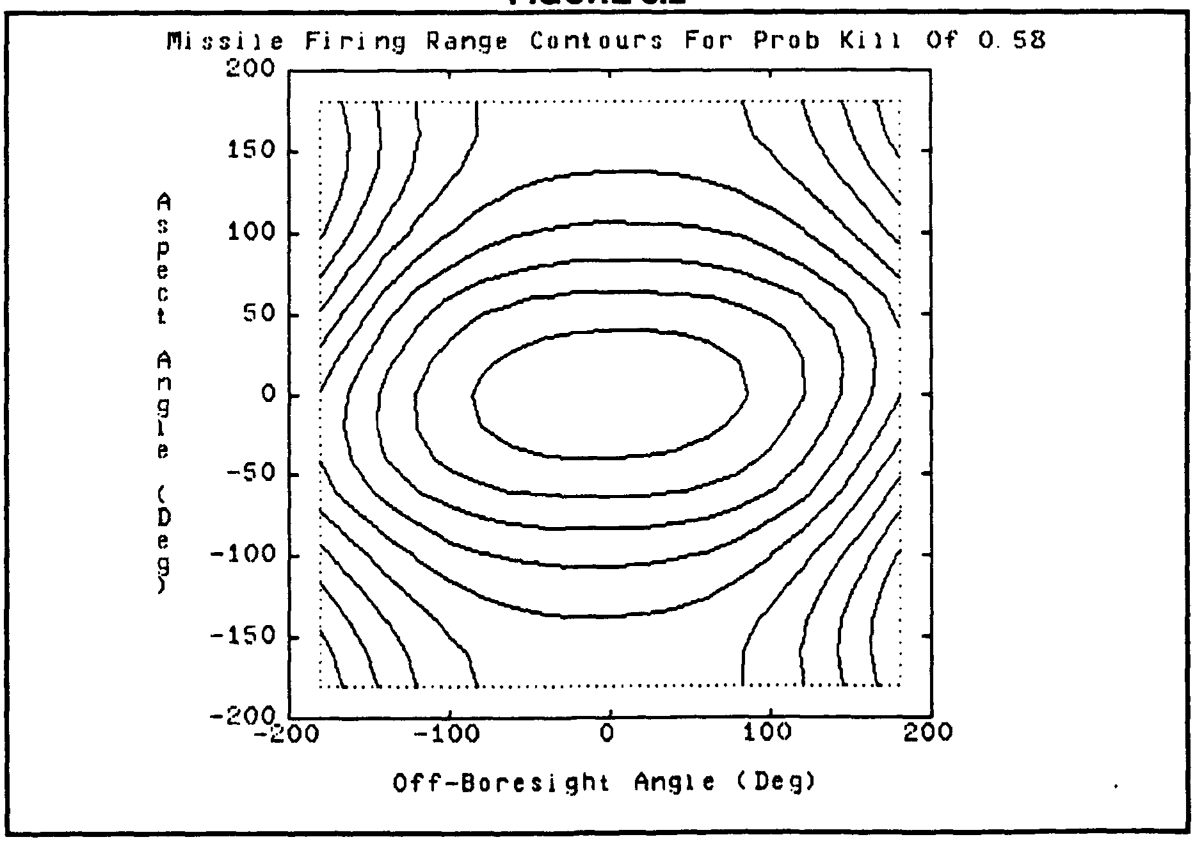


FIGURE 5.2



The avoidance surface is determined by the amount of risk that a pilot or the pilot's superiors are willing to accept. We will assume for simplicity that it is desirable to keep the probability of being destroyed by an adversarial missile launch below a predetermined threshold. Equations (4.1.53) and (4.1.56) are used again, this time to determine the avoidance surface. The role of aircraft and missile parameters are now interchanged to indicate ownship aircraft capabilities versus the adversarie's missile capabilities. Figures 5.3 and 5.4 show an avoidance surface for an adversary missile kill probability of 10% percent when Vm = 1, Rm = 1, Va = .33, Ra = 2, Tbt = 7, $\alpha = 0.75$, $\beta = 0.7$, and $P_D = 0.4$.

FIGURE 5.3
Missile Range Avoidance Surface For Prob Kill Of 0.10

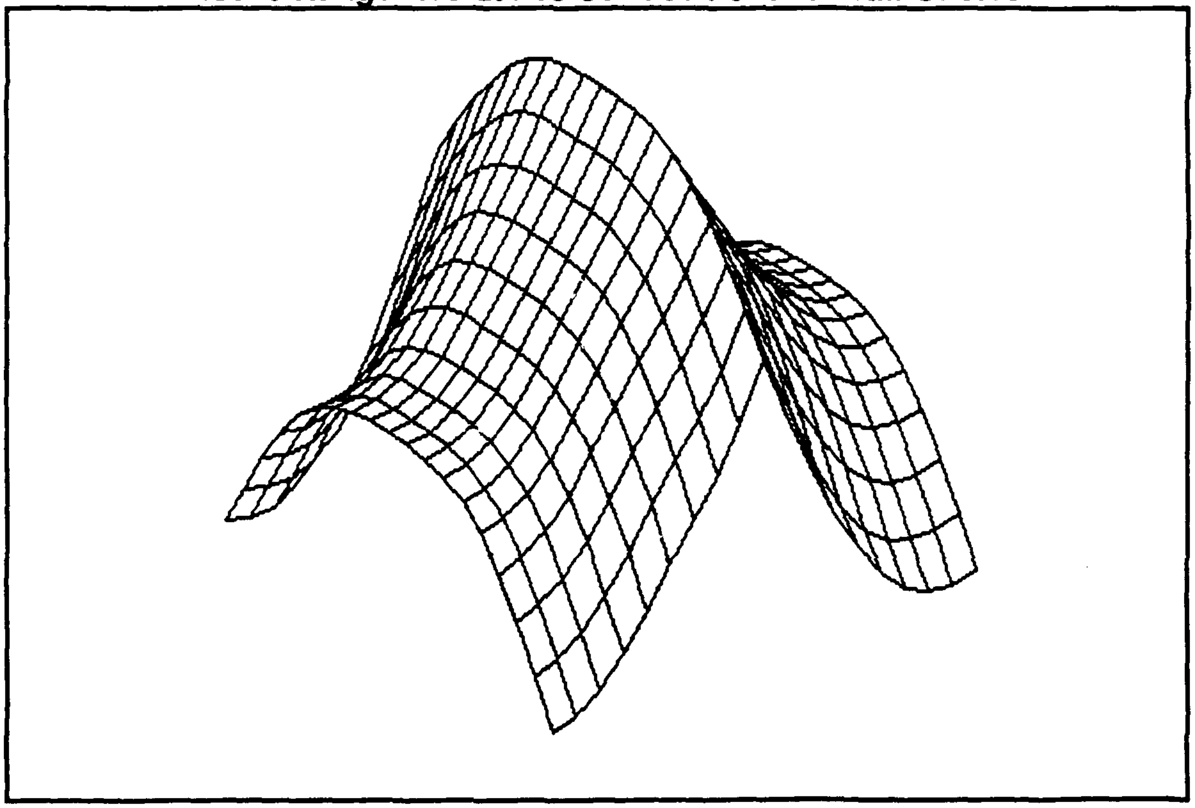
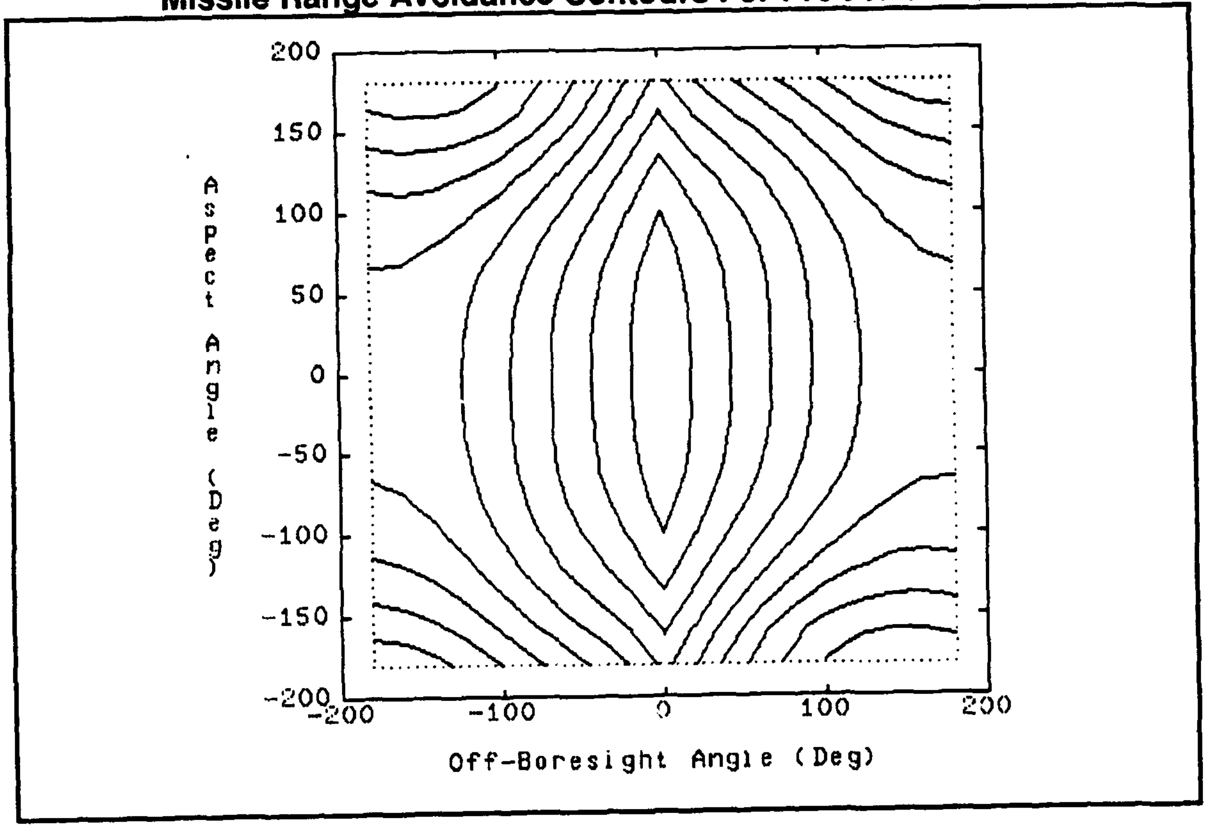


FIGURE 5.4
Missile Range Avoidance Contours For Prob Kill Of 0.10



5.2 GOAL SELECTOR

Once the firing and avoidance surfaces are determined, the problem becomes one of determining trajectories and controls that allow the ownship aircraft to reach the firing surface without penetrating the avoidance surface. The goal selector of the semantic control paradigm must identify the differential game, parameters and role that the ownship aircraft should assume from the knowledge base of differential games. Three alternatives for the system goal selector expert system will be explored, rule based, neural nets, and The Analytical Hierarchy Process.

One key requirement for the "expert system" implementation of the goal selector is that the knowledge (i.e. the differential game library) must be separated from how the knowledge is manipulated. It is imperative that the capability exist to add new differential games to the knowledge base for the possibility of improved performance, without modifying the system as a whole. This feature is known as "incremental development"[34]. "Incremental development" allows the maintenance and improvement of performance by incrementing the knowledge base as more is learned about ownship and adversarial performance via the models incorporated into the knowledge base. Without "incremental development" a new model, or improved adversarial performance could mean the obsolescence or complete redesign of the semantic control system.

A "shell" is a tool for building a limited set of knowledge base applications based on generalizing the requirements common among these applications. The

[34]Lazarev, G. L. pg 134

"shell" serves to separate the knowledge represented and the "inference engine". The "inference engine" is primarily responsible for control. The "inference engine" must also handle features such as user interaction, explanation capabilities, and uncertainty. Our goal is to outline how to build the expert system "shell" for the goal selector such that "incremental development" is supported, and performance will approach optimality for the current differential games in the knowledge base.

5.2.1 RULE BASED APPROACH

The first methodology to be considered for building the differential game expert "shell" will be a rule based system. It should be noted that the majority of existing expert systems are rule based due to their naturalness, ease, and uniformity of expression. Rule base systems are of the form

If *condition(s)* then *action(s)*

These rules can be used in a forward or backward chaining fashion. Forward chaining starts with a rule base and a set of known facts. A rule that matches the current set of known facts is triggered. The set of known facts is modified by the triggered rule's action. The process is repeated for the set of updated facts until a solution, or a failure is reported. Reasoning is performed from facts, using rules until a solution is found.

Backward chaining is goal driven processing. Reasoning is performed from a goal using rules to existing facts. Rules are used from actions to conditions. Each hypothesized goal is reduced to a set of sub-goal. The process is then repeated until the reduced sub-goal can be identified with the existing facts. One of the main uses of backward chaining is the generation of explanations for a decision.

There are two reasons we give only brief consideration to the rule based approach. Generating general rules splicing the differential games together appears to demand understanding the dynamics of the encounters at the same level that prevents the solution of higher order games. Rule based approaches also tend to be brittle. An increment in the knowledge base could change a set of complex rules drastically. Generating a set of rules for a specific set of differential games would create a non-robust goal selector. Rule based approaches do not appear promising.

5.2.2 NEURAL NET APPROACH

The next expert system type to be considered is the Artificial Neural Net expert system as proposed by Gallant. Interest in Artificial Neural Nets has had extremes in popularity since the early 1960s. Artificial Neural Nets are computational mechanisms based on the structure of the brain. Currently there is an explosion of interest and research in Artificial Neural Nets and their capabilities. This explosion follows a long winter of interest in Artificial Neural Nets during the 1970s.

Initially, during the 1960s, there was a great deal of excitement over linear neuron-like models termed perceptrons[35]. Minsky and Papert demonstrated in their book Perceptrons the key limitation that perceptrons could only solve linearly separable problems. This revelation lead to the wide scale abandonment of Artificial Neural Nets in the 1970s and to the flow of research dollars to

[35]Rosenblatt

Artificial Intelligence (AI). Recent advances with multilayered networks and new network learning paradigms such as backpropagation have overcome the limitation of linear separability and caused a major resurgence in artificial neural net research.

Hecht-Nielsen[36] gives the following general definition of an artificial neural net system [ANS]:

> A neural network is a parallel, distributed information processing structure consisting of processing elements (which can possess a local memory and carry out localized information processing operations) interconnected together with unidirectional signal channels called connections. Each processing element has a single output connection which branches ("fans out") into as many collateral connections as desired (each carrying the same signal - the processing element output signal.) The processing element output signal can be any mathematical type desired. All of the processing that goes on within each processing element must be completely local; i.e., it must depend only upon the current values of the input signal arriving at the processing element via impinging connections and upon values stored in the processing element's local memory.

Simpson[37] gives a simpler, less rigorous definition, "... an ANS is a nonlinear directed graph with edges that is able to store patterns by changing the edge weights and is able to recall patterns from incomplete and unknown inputs".

A biological neuron is the basic building block of the nervous system. Figure 5.5 shows a simplified view of a biological neuron. The body cell of the neuron is termed the "soma". Connected to the "soma" are multiple "dendrites" and an "axon", these serve as the mechanism of communication to other neurons. The "dendrites" are spine like connections which receive stimulus from other neurons. Each neuron has a single "axon" that serves to transmit the same

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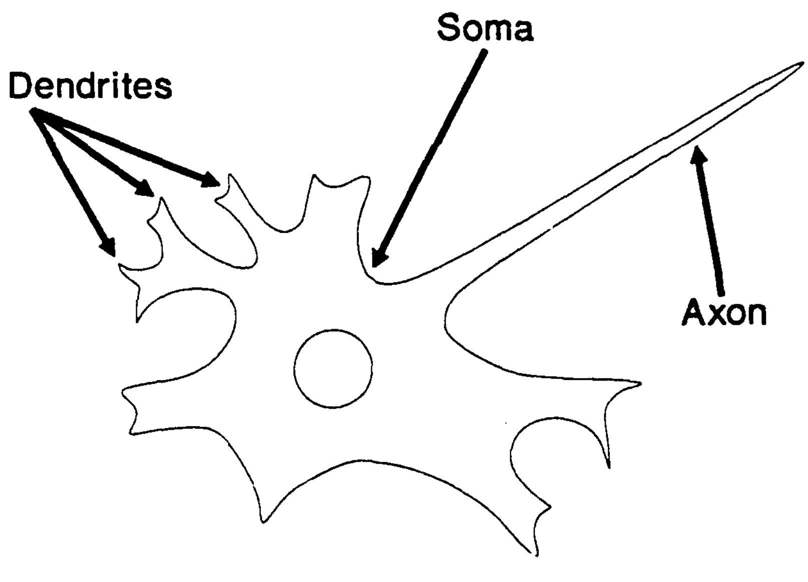
[36]Simpson pg 3

[37]Simpson pg 4

stimulus to all the other connected neurons. The connection between a neuron's "axon" and another neuron's "dendrite" is termed a "synapse". Each "synapse" has a level of transmission of the stimulus on the "axon" to the connected "dendrite". Only when a neuron receives sufficient stimulus from other neurons connected at the "dendrites" does the neuron become active and send a stimulus out on its own "axon". Since each neuron accepts a different level of stimulus from a connected neuron based on the transmission level at the "synapse", the network knowledge is equivalent to these transmission levels. Biological neural networks work by a biochemical process that is beyond our scope of interest here.

FIGURE 5.5

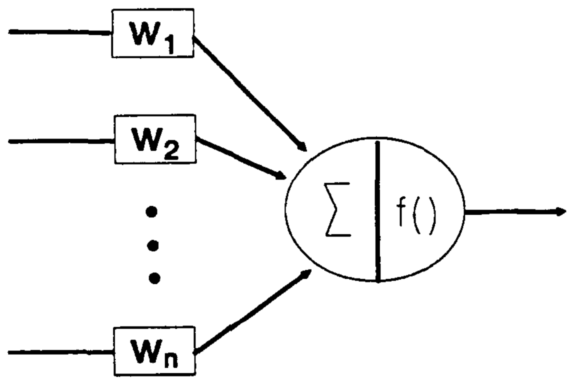
Biological Neuron



The basic model for an artificial neuron is shown in Figure 5.6. Lines (or wires) replace the functions of biological "axon" and "dendrites". "Synapses" are replaced by weights (or resistors). The body of the neuron or "soma" is split into two components. The first component is an adder that sums up the weighted outputs of other neurons. The second component is the activation function. This function determines the artificial neuron's activation level based on weighted net input. Typically this function is an s-shaped function known as a squashing function.

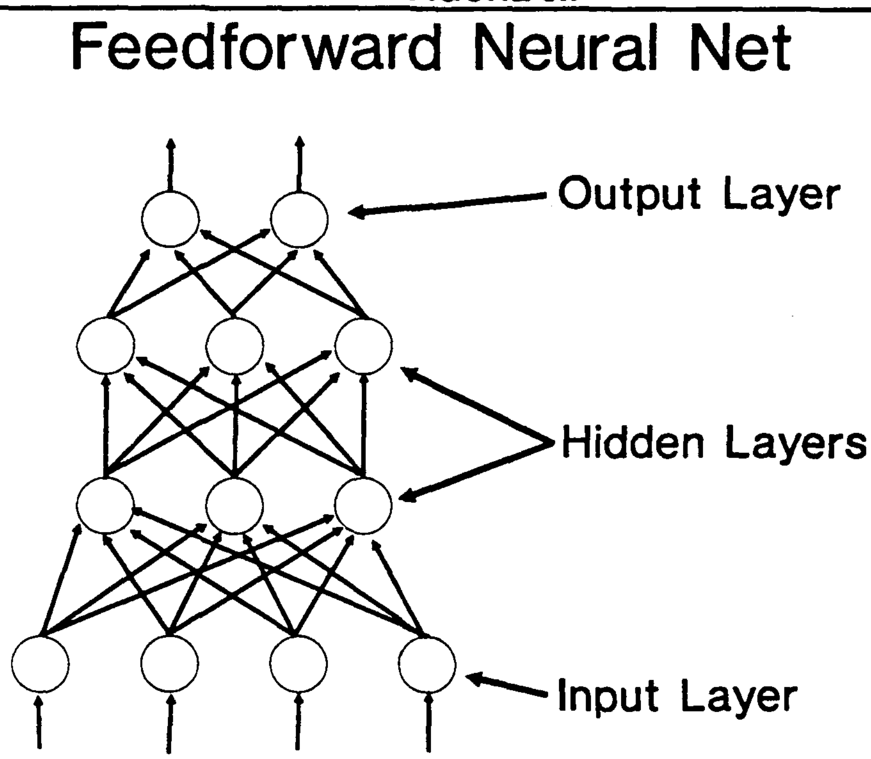
FIGURE 5.6

Artificial Neuron Model



The method of connecting the network neurons defines the network topology. Two general categories for networks exist, recurrent and non-recurrent. Recurrent networks have cycles in the network connections. These cycles make recurrent network dynamical systems. Our interest lies in non-recurrent networks. Non-recurrent networks have no cycles. A non-recurrent network can be viewed as a transformation from n-dimensional euclidean space to m-dimensional euclidean space. The special instance of a non-recurrent net with which we concern ourselves is the feedforward net shown in Figure 5.7.

FIGURE 5.7



A feedforward net consists of a number of layers of neurons. Neurons in one layer are restricted to outputting only to neurons in the next layer, and inputting only from neurons in the previous layer. There is always an input layer which simply serves to buffer the inputs. An output layer also exists and its activation activation function f() can be linear or nonlinear. There can also be a number of hidden layers. These hidden layers lie between the input and output layers and have a non-linear activation function f(). The power of the feedforward net lies in the non-linear nature of the hidden layers. Recent research has shown that any measurable function can be approximated almost everywhere by a feedforward net with one hidden layer.[38] The network connection weights can be programmed a priori. The weights are more commonly learned recursively. Different learning paradigms can be found in Simpson (1989).

Gallant has demonstrated a methodology to use artificial neural networks to build classification expert systems where the outputs from the system can be represented by variables that take on values from a finite set. Each variable has its own finite set of possible values. The particular artificial neural network implementation for our interests will have a Boolean (two value) classification system. Classification expert systems can handle a wide variety of applications. Suppose a particular variable X can take on continuous values in the range [0,1], X could be approximated by several Boolean choice variables X_1, X_2, X_3, where each X_i corresponds to a range of values:[39]

if $X \geq 1/4$, then $X_1 = 1$; else $X_1 = -1$;

if $X \geq 1/2$, then $X_2 = 1$; else $X_2 = -1$;

if $X \geq 3/4$, then $X_3 = 1$; else $X_3 = -1$;

so that $X = .6$ would correspond to $X_1 = X_2 = 1$, $X_3 = -1$.

The artificial neural net for the classification expert system shall have the architecture described below. The net architecture shall be feedforward with one hidden layer. The neurons are perceptrons having discrete activation levels, -1, 0 or 1, corresponding to the Boolean values False, Unknown or True

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[38]Hornik

[39]Gallant pg 154

respectively. The neuron activation function relating the activation of neurons (u^{k-1}_j) on layer k-1 to neurons (u^k_i) on layer k is given by:

$$net^k_i = \sum_j w^k_{ij} u^{k-1}_j$$

$$u^k_i = \begin{cases} 1, & \text{if } net^k_i > 0 \\ 0, & \text{if } net^k_i = 0 \\ -1, & \text{if } net^k_i < 0, \end{cases}$$

where w^k_{ij} is the weight connecting neuron i on layer k to neuron j on layer k-1.

The name of each neuron corresponding to each variable of interest must also be specified. The variables of interest must also be partitioned into dependent and independent classes. The independent variables of interest are assigned to the input layer, while the dependent variables are assigned to the output layer.[40] Partial knowledge about the relation between dependent and independent variables can be directly incorporated in the net by allowing only partial connectivity through the hidden layer between sets of input and output neurons. The values of the weights can be directly programmed if the relations are actually known, or more realistically they can be learned from a training set of input and output pairs.

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[40]This partitioning differs slightly from that proposed by Gallant. Gallant's method was to list all the dependencies (adjusting to avoid cycles) having no particular input, output, and intermediate layers. The number of layers corresponded to the minimum number of flow forward dependencies. (Gallant pg 156)

Our interest in classification systems will be to determine what models to use, and the particular parameterization of the models chosen. The problem is divided up hierarchically. One classification network is designed to determine what differential game and models should be used, along with pursuer and evader role assignments. Another set of classification nets is also specified, one net for each output combination from the first net. A net that corresponds to an output combination determines the parameterizations and flight characteristics for the model, differential game, and role chosen.

Figure 5.8 illustrates the classification net that determines which differential game from the knowledge base (currently in our stage I simulator) will be chosen along with some of the criteria that might be used to make the decision. Figure 5.9 illustrates the net corresponding to the "Harrier Pursuit" output of the differential game classification net shown in Figure 5.8. The net shown in Figure 5.9 determines the parametrization of the F16 pursuit of Harrier differential game. A game parameterization classification net exists corresponding to each of the output classifications of the net shown in Figure 5.8.

The question of determining the weights of the classification nets discussed above and shown in Figures 5.8 and 5.9 still remains open. Generally the connection weights are learned recursively via training by one of the methods discussed in Simpson. Training is generally done from a training set of desired input output matchings for asymptotic cases. The goal is to learn the unknown underlying mapping from input to output space so that correct outputs will be put forth for inputs not in the training set.

FIGURE 5.8

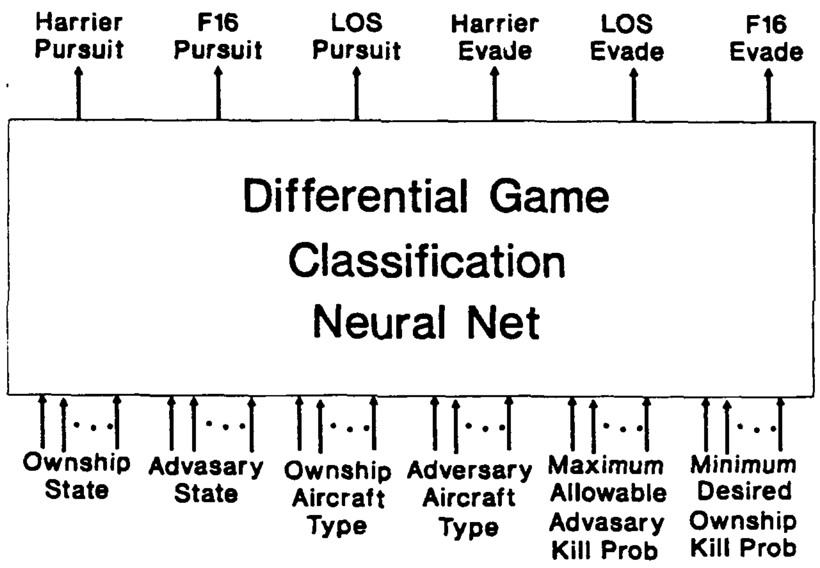
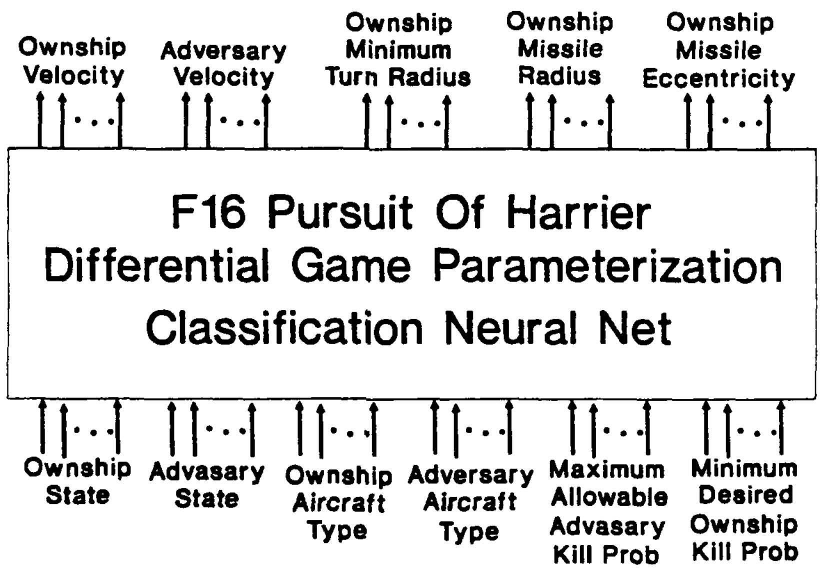


FIGURE 5.9



The generation of a representative training set is typically non-trivial. Training sets must be generated for the differential game classification neural net, and for each of the individual parametrization classification neural nets. Two methods can be considered for the generation of the training sets: analysis and simulation.

Generating the training set by analysis shares the main difficulty of the rule based methodology: the requirement for complete mathematical understanding of how all the differential games perform relatively for each member of the training set.[41] It may be possible to generate analytically members of the training set for a few asymptotic cases, but the their generation for every case by this methodology is not really practical.

Simulation methodologies appear to be the most promising means of generating a training set. Each of *the differential game parametrization* classification neural nets can be trained independently by closing the loop simulating forward, and adjusting the weights by propagating the errors backward in time. Once each of the individual differential game parametrization classification neural nets is trained, the differential game choice classification neural net (Figure 5.8) training set can be generated via comparative simulation. This methodology involves the generation of the appropriate neural net output for a given neural net input by searching all possible output via simulation for the best performance. Extensive search via simulation would of course be extremely computationally intensive. Therefore heuristic search methods might offer improvement in search speed. The neural nets once trained on a large enough

[41]It is this difficulty the prevents the generation of solutions to higher order differential games.

set, would be expected to generalize well to cases not directly included in the training set.

One important requirement is that the expert system be able to justify its decisions to the user. A system to aid pilots must be able to explain decisions to the pilot to find acceptance. Gallant gives a methodology to generate explanations of decisions via the generation of if-then rules from classification neural nets.[42] His methodology is as follows:

i) List all inputs that have contributed positively to the discriminant of Z_j^k. Figure 5.10 generates Z_1^{k-1}, Z_2^{k-1}, and Z_5^{k-1}.

ii) Arrange the list by decreasing absolute value of the weights. Figure 5.10 gives Z_2^{k-1}, Z_5^{k-1}, and Z_1^{k-1}.

iii) Generate clauses for an if-then rule from the ordered list from ii) until

$$\left[\sum_{Z^k_i \text{ used for clause}} |W^k_{ij}| \right] > \left[\sum_{\text{remaining } Z^k_i} |W^k_{ij}| \right]$$

is satisfied. The generated if-then rule for Figure 5.10 given by this methodology is:

if Z_2^{k-1} is False and Z_5^{k-1} is True

then Conclude That Z_j^k is True

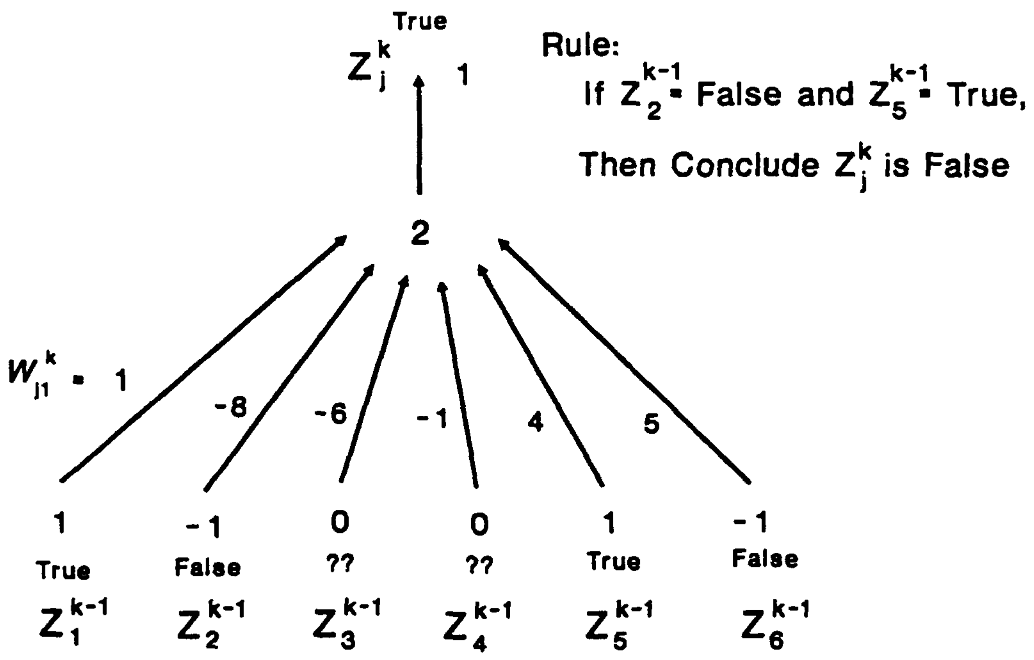
[42]Gallant pg 163

Gallant's methodology generates as set of if-then rules for layer to layer connections that can be used in a backward-chaining fashion to explain decisions.

In conclusion we has shown in this section that artificial neural nets are candidates for implementing the semantic controller identifier for air-combat. Such a methodology would provide a real-time mechanism for determining what differential games and parameters should be chosen from the knowledge base. The main drawback of this approach is the difficulty in obtaining training sets.

FIGURE 5.10

Explanations by if-then Rules



5.2.3 ANALYTICAL HIERARCHY APPROACH

The final approach to be discussed for implementing the identifier is the Analytical Hierarchy Process originated by T.L. Saaty of the Wharton School. The Analytic Hierarchy Process (AHP) is a general theory of measurement that can be used in multicriteria decision making. Ratio scales are derived from paired comparisons. To use AHP in a decision problem, a hierarchic structure that represents the problem is needed, along with pairwise comparisons to establish relations within the structure.

Figure 5.11 shows an example of a hierarchic structure for a simple example of choosing the best high school for a student to attend. This simple hierarchy has the overall goal as the apex (choose the school that gives the most overall satisfaction). The criteria for selecting falls on the second layer. The lowest layer contains the alternative (the school) choices. More complex decision problems that have criteria and sub-criteria would have more layers in the hierarchy. Figure 5.12 shows the pairwise comparison of the importance of the criteria with respect to satisfaction with school. The relative importance of each of the criteria can be shown to be the principal eigenvector of the matrix of pairwise comparisons. Pairwise comparison would be repeated for each school choice with respect to each of the criteria. The set of principal eigenvectors for each of the comparison matrices would finally be composed via matrix multiplication to give an overall ranking of the merits of each alternative (school choice).

The AHP has a formal mathematical basis. We first present some partially ordered set notation.

Definition 5.1: Let S be a partially ordered set (P.O.S). Then for $x,y \in S$:

i) $x < y$ implies $x < y$ and $x \neq y$

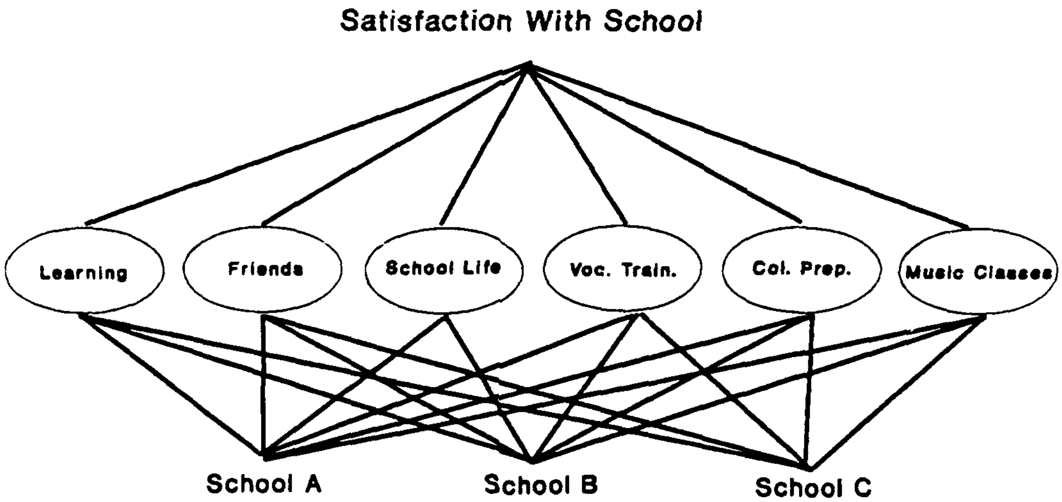
ii) y covers x, if $x < y$ and there does not exist $t \in S$ such that $x < t < y$

iii) $x^- = \{ y \mid x \text{ covers } y\}$

iv) $x^+ = \{ y \mid y \text{ covers } x\}$

FIGURE 5.11

HIERARCHY FOR HIGH SCHOOL CHOICE



Developed For Saaty's Son

FIGURE 5.12
Comparison of characteristics with respect to overall satisfaction with school

	Learning	Friends	School Life	Vocation Training	College Prep.	Music Classes
Learning	1	4	3	1	3	4
Friends	1/4	1	7	3	1/5	1
School Life	1/3	1/7	1	1/5	1/5	1/6
Voc Train	1	1/3	5	1	1	1/3
College Prep	1/3	5	5	1	1	3
Music Class	1/4	1	6	3	1/3	1

Given our P.O.S notation a Hierarchy can formally be defined. The following definition introduces the concept of layers for a P.O.S.

Definition 5.2: Let H be a finite P.O.S., H is a hierarchy if the following conditions are satisfied;

i) There exist a partition of H into set L_k, $k = 1,...,h$ where $L_1 = \{b\}$, b is a single element (apex).

ii) $x \in L_k$ implies x^- is a subset of L_{k+1}, $k = 1,..., h-1$

iii) $x \in L_k$ implies x^+ is a subset of L_{k-1}, $k = 2,..., h$

We now lay out the ground work for determining rankings of the elements of a layer with respect to each member of the next higher layer. Let *U* be a finite set of "alternatives". Let *C* be a set of criteria with respect to which the elements in *U* are compared. For $c \in C$, let P_C: $U \times U \rightarrow R^+$ such that $P_C(A_i,A_j) \equiv a_{ij} \in R^+$ for every $A_i,A_j \in U$.[43] The function P_C is of course the pairwise comparison.

Definition 5.3: For every $c \in C$, $(U \times U, R^+, P_C)$ is called a "primitive scale".

[43] R^+ is the set of positive reals

<u>Definition 5.4</u>: For $A_i, A_j \in U$ and $c \in C$:

i) $A_i >_C A_j$ iff $P_C(A_i,A_j) > 1$ "A_i dominates A_j wrt c"

ii) $A_i \approx_C A_j$ iff $P_C(A_i,A_j) = 1$ "indifference"

Reciprocal consistency is imposed on the pairwise comparisons. It is imposed so that if A is twice as important as B, then B is half as important as A. Saaty's first axiom states this condition.

<u>Axiom 5.5</u>: For every $A_i, A_j \in U$ and $c \in C$, $P_C(A_i,A_j) = 1/P_C(A_j,A_i)$.

Our main sub-objective at this point is this: given a matrix $A = (a_{ij}) = (P_C(A_i,A_j))$ of pairwise comparisons obtain a scale of rank order of the alternatives. Let $R_{M(n)}$ be the set of (nxn) positive reciprocal matrices, and define a function $W:R_{M(n)} \rightarrow [0,1]^n$ which maps (nxn) reciprocal matrices to n-dimensional vectors with positive components with magnitude less than or equal to one. The function W determines from a matrix of pairwise comparisons the relative rankings of the alternatives based on the relative magnitude of the corresponding components of the vector.

<u>Definition 5.6</u>: $(R_{M(n)},[0,1]^n,W)$ is the "derived" scale.

There are of course many different "derived" scales. Saaty built his "derived" scale around the notion of consistency. Consistency is the property of transitivity in the pairwise comparisons.

<u>Definition 5.7</u>: The mapping P_C is said to be consistent iff,

$P_C(A_i,A_j)P_C(A_j,A_k) = P_C(A_i,A_k)$ for every i,j,k

or

$a_{ij}a_{jk} = a_{ik}$ for every i,j,k

While the pairwise comparisons are not required to be totally consistent (they seldom are), the "derived" scale is based on consistency for the idealized case. Small perturbations in consistency can be likened to noise in a measurement. Letting w_i be the i^{th} component of the derived scale vector W, then if A were totally consistent then $a_{ij} = w_i/w_j$ for every i,j = 1,2,...,n. This implies the following:

$$a_{ij} \frac{w_j}{w_i} = 1$$

$$\sum_{j=1}^{n} a_{ij}w_j/w_i = n$$

$$\sum_{j=1}^{n} a_{ij}w_j = nw_i$$

$$AW = nW$$

Thus W is the eigenvector corresponding to the maximum eigenvalue n. Small perturbations of consistency implies by the continuity of the eigenvalues and eigenvectors that:

$$AW = \Gamma_{max}W,$$

where Γ_{max} is the maximum eigenvalue of A, and W is the corresponding eigenvector.

At this point we have shown how Saaty's methodology relates alternatives on one layer of the hierarchy to each of the criteria on the preceding layer. It still is to be shown how to relate alternatives to criteria on layers not directly adjacent, in particular to relate alternatives on the bottom layer to the goal on the apex of the hierarchy.

Recall the notation of definitions 5.1 and 5.2. Assume that the "derived" scales (normalized principal eigenvectors corresponding to the pairwise comparison matrixes) have been defined for every member of the hierarchy ($x \in H$) ranking all member of the next level of the hierarchy (X^-) with respect to x, i.e., for every $x \in H$ a function,

$$W_X : X^- \rightarrow [0,1] \text{ such that } \sum_{y \in X^-} W_X(y) = 1,$$

is defined. Given any element $x \in L_\alpha$ of a particular level α of the hierarchy, and subset S of a lower level L_β ($\alpha < \beta$), Saaty defined a function $W_{x,s}:S \rightarrow [0,1]$ which reflects the properties of the "derived" scales of the intermediate levels L_K, $k = \alpha,\ldots,\beta-1$. Let $X = \{x_1,\ldots,x_{mk+1}\} = L_{K+1}$; $Y = \{y_1,\ldots,y_{mk}\} = L_K$, and assume there exist $z \in L_{K-1}$ such that $Y = z^-$, then the function $W_{z,X}:X \rightarrow [0,1]$ relating hierarchy layer L_{K-1} to layer L_{K+1} can be constructed as follows:

$$W_{z,X}(x_i) = \sum_{j=1}^{m_K} W_{yj}(x_i) W_Z(y_j);\ i = 1,\ldots,m_{K+1}.$$

The matrix B is formed by letting $b_{ij} = W_{yj}(x_i)$, defining $W_i = W_{z,X}(x_i)$ and $W'_j = W_z(y_j)$ then:

$$W_i = \sum_{j=1}^{m_K} b_{ij} W'_j\ ;\ i=1,\ldots,m_{K+1}$$

$$W = BW'$$

This derivation relates layer k-1 to layer k+1 by a simple matrix multiplication, and is generalized in the following theorem.

<u>Theorem 5.8</u>: Let H be a hierarchy with largest element b and h levels. Let B_K be the priority matrix of the k^{th} level, $k = 1,\ldots,h$, if W' is the priority vector of the p^{th} level with respect to some $z \in L_{p-1}$, then the priority vector W of the q^{th} level ($p < q$) with respect to z is:

$$W = B_q B_{q-1} \ldots B_{p+1} W',$$

in particular with respect to b

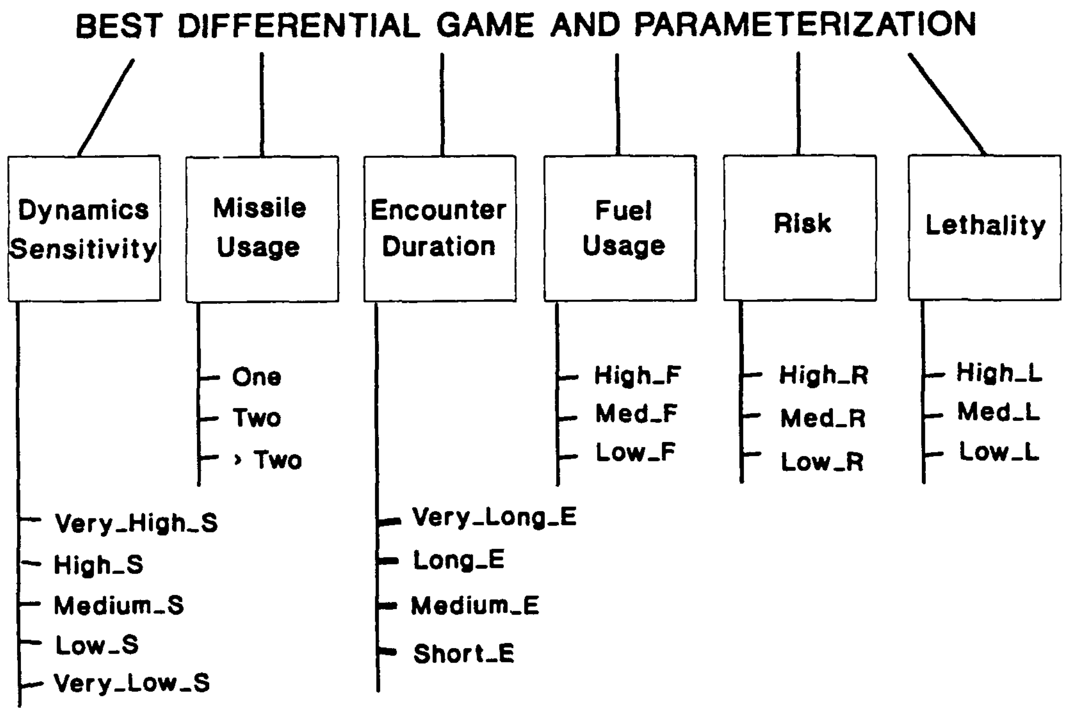
$$W = B_h B_{h-1} \ldots B_2 b_1$$

The power of the AHP has been shown to be the structure it brings to multi-criteria decision problems, along with the necessity to make only pairwise comparisons among alternatives. It is much easier to compare two objects at a time, then to rate a whole group. The AHP mechanism has the power to compose these local rankings into the hierarchic structure and produce an overall relative ranking of the merits of the bottom level alternatives with respect the overall goal.

Our interest is to incorporate the AHP as a mechanism to implement the system identifier. It is our goal here to point out only how the identifier might be formulated to use the AHP, along with projected strengths and weakness.

A hierarchy is created indicating criteria and sub-criteria for choosing the most appropriate differential game and parameterization. Figure 5.13 shows a sample three layer hierarchy for choosing the differential game. The criteria in this example include fuel usage, missile usage, lethality of ownship against adversary, risk of ownship from adversary, duration of the encounter, and dynamic sensitivity of the differential game model. Dynamic sensitivity of the differential game model measures how much extra headroom a model has to the dynamics of the adversary before the model is invalidated by penetration of the barriers or other critical assumptions of the underlying model are invalidated.

FIGURE 5.13
EXAMPLE AHP HIERARCHY FOR CHOOSING MOST APPROPRIATE DIFFERENTIAL GAME AND PARAMETERIZATION



A pairwise comparison matrix is continually updated for each layer of the hierarchy with respect to each of the criteria of the previous layer. One modification to the use of the AHP is made here by following the method of Forman (1987). Instead of directly comparing the differential game alternatives, levels are designated for each of the bottom layer criteria. An example of these levels is shown in Figure 5.13.

A comparison matrix for the levels is created only with respect to the criteria for which the levels were designated, enabling a priority vector to be determined. A priority vector relating all the levels to a particular criterion can be created by augmenting the original priority vector with zeroes for non-related levels. Figure 5.15 illustrates the comparison matrix and priority vectors for the fuel levels of Figure 5.13. Once the augmented comparison vectors have been determined with respect to each bottom level criterion, the overall priority vector of the levels can be determined by the normal AHP composition technique of Theorem 5.8. Figure 5.16 shows an example of the priority vectors for the levels of each criterion, the priority vector of the criteria, and the overall composed priority vector of the levels with respect to the goal of choosing the best differential game. The pairwise comparison matrices were omitted for brevity.

A scoring technique is used to choose the differential game and parameterization from the knowledge base. A level under each criterion is determined for each differential game alternative. The score consists of the sum of the weights corresponding to the designated levels. Assume for our simple example that there are two differential games and each game has two parameterizations which have had levels determined for the criteria as shown below. The respective scores using the values of the overall priority vector of Figure 5.16 are also determined and are shown in Figure 5.14.

Differential Game 1 Parametization 2 has the highest score and would be the differential game chosen. It is important to note that the criteria priority vectors, pairwise comparisons, and levels would be dynamically changing during an air-combat mission. Fuel might have a low priority early in a mission, but later might have high priority, once a shortage develops.

FIGURE 5.14
EXAMPLE SCORING

Diff Game 1 Param 1 (Score: 0.375)

High_S Two Short_E Low_F Low_R Med_L

0.035 + 0.020 + 0.043 + 0.045 + 0.196 + 0.036 = 0.375

Diff Game 1 Param 2 (Score: 0.434)

Los_S One Med_E Med_F Med_R High_L

0.138 + 0.080 + 0.023 + 0.023 + 0.098 + 0.072 = 0.434

Diff Game 2 Param 1 (Score: 0.335)

Very_High_S One Med_E Low_F Med_R High_L

0.017 + 0.080 + 0.023 + 0.045 + 0.098 + 0.072 = 0.335

Diff Game 2 Param 2 (Score: 0.316)

Medium_S >Two Long_E Low_F Low_R Low_L

0.035 + 0.010 + 0.012 + 0.045 + 0.196 + 0.018 = 0.316

FIGURE 5.15
Level Comparisons With Respect To Fuel Usage

	High_F	Medium_F	Low_F
High_F	1	1/2	1/4
Medium_F	2	1	1/2
Low_F	4	2	1

"Priority Vector Of Fuel Usage Levels"

High_F	0.143
Medium_F	0.286
Low_F	0.571

"Augmented Priority Vector Of Levels WRT Fuel Usage"

Very_High_S	0.0
.	.
.	.
Medium_E	0.0
Short_E	0.0
Low_F	0.571
Medium_F	0.286
High_F	0.143
High_R	0.0
Medium_R	0.0
Low_R	0.0
.	.
.	.
Low_L	0.0

FIGURE 5.16
Priority Vectors For Example Hierarchy

"Priority Vector Criteria With Respect To Goal Of Choosing Most Appropriate Differential Game"

Dynamic Sensitivity	0.259
Missile Usage	0.110
Encounter Duration	0.084
Fuel Usage	0.079
Risk	0.343
Lethality	0.125

"Priority Vector Of Dynamic Sensitivity Levels"

Very_High_S	0.067
High_S	0.133
Med_S	0.267
Low_S	0.533

"Priority Vector Of Missile Usage Levels"

One	0.727
Two	0.182
> Two	0.091

"Priority Vector Of Encounter Duration Levels"

Very_Long_E	0.074
Long_E	0.138
Medium_E	0.275
Short_E	0.513

"Priority Vector Of Fuel Usage Levels"

High_F	0.143
Medium_F	0.286
Low_F	0.571

"Priority Vector Of Risk Levels"

High_R	0.143
Medium_R	0.286
Low_R	0.571

"Priority Vector Of Lethality Levels"

High_L	0.571
Medium_L	0.286
Low_L	0.143

"Overall Priority Vector Of Criteria Levels With Respect To Goal Of Choosing Most Appropriate Differential Game"

Very_High_S	0.017
High_S	0.035
Med_S	0.069
Low_S	0.138
One	0.080
Two	0.020
> Two	0.010
Very_Long_E	0.006
Long_E	0.012
Medium_E	0.023

Short_E	0.043
High_F	0.011
Medium_F	0.023
Low_F	0.045
High_R	0.049
Medium_R	0.098
Low_R	0.196
High_L	0.072
Medium_L	0.036
Low_L	0.018

An advantage of the AHP implementation methodology for the system identifier is that incremental development is easily supported. New games and parameterizations can easily be incorporated by simply determining the appropriate level under each criterion, and including the new games to the scoring. An explanation of the decision is also available by examining the different layer weights with respect to the goal of choosing the best differential game. Risk (0.343) and Dynamic Sensitivity (0.259) are the most important criteria for the example. Diff Game 1 Param 2 (0.434) had the best level value for Dynamic Sensitivity, and was as good as all other choices for Risk.

One area of difficulty that still requires research for the AHP based method is how to determine levels from the differential game and parameterization when the criteria are not part of the game formulation. The appropriate criteria, sub-criteria, and proper number of levels for each criterion also require further investigation.

5.3 CONTROL ADAPTER

The control adapter is responsible for following the differential game trajectories from the goal adapter, and for firing missiles (reproduction). In our current implementation the goal adapter is a rule based expert system. The goal adapter determines the aircraft controls that best follow the differential game trajectory. Techniques for following a known trajectory are not new and simply require tradeoffs when a trajectory requires controls outside the aircraft flight envelope.

The phase 1 simulator currently has implemented an expert system trajectory follower. It determines how to use excess capabilities when available. Suppose that we use a two dimensional differential game (F16 pursuit of Harrier), the controller may find that the turn called for is wide enough that excess thrust and lift are available. The controller will make the simpler turn called for by the two-dimensional game and use the excess flight capabilities to climb or dive to bring the ownship and adversary into the same horizontal plane to better fit the differential game model. These ideas are implemented currently in the control adapter with specific rules dependent upon the differential game model chosen. The control adapter also determines if a missile launch opportunity has occurred by reaching the terminal manifold or the differential game, or a firing surface of the probabilistic model has been reached.

6. DIFFERENTIAL GAME KNOWLEDGE BASE

This section will cover the incorporation of differential games into the knowledge base. Currently our phase I simulator has strategies of a Harrier chasing an F16 (highly maneuverable but slower pursuer), an F16 chasing a Harrier (faster pursuer against highly maneuverable evader), and line of sight strategies (LOS). The LOS strategies consist of turning at the maximum rate available to the aircraft while maintaining a minimum desired velocity. There are many implementations of this strategy currently available such as proportional navigation (PRONAV), and LOS will not be extensively discussed here. The other four strategies (a pursuit, and an evasion strategy for each game) are extensively discussed and the game of degree solutions are new for eccentric terminal manifolds.

Two obstacles prevent the building of a large knowledge base of differential game solutions. The first one deals with dimensionality and discontinuity of trajectories and controls. This obstacle has prevented the generation of solutions to all but very simple low dimensional problems. The second obstacle is the need for feedback control laws that can be generated in real time. The normal procedure is to solve a corresponding two point boundary value problem (TPBVP) numerically. This numeric solution cannot be found in real time and is the open loop control. Open loop control does not allow one to take advantage of an opponent's mistakes.

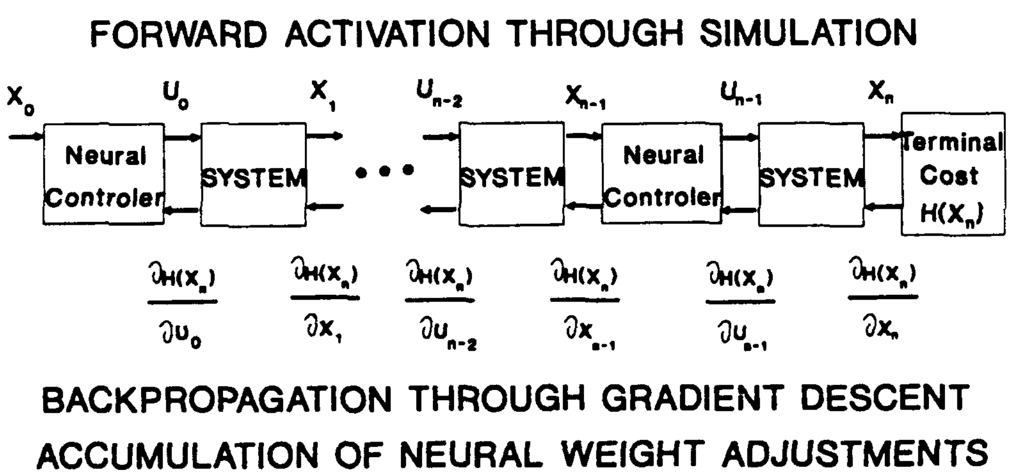
Currently research in McDonnel Douglas Missile Component Company's guidance navigation and control technology group is exploring the use of Neural Nets to overcome the obstacles to the use of differential game technology. A brief overview of Neural Nets is found in section 5.2.2.

The approach being taken is to train individual Neural Nets to be the optimal feedback controllers for the pursuer and evader. The only information assumed to be available is the initial state of the system, and the final state of the system via simulation. No knowledge of the true optimal trajectories or feedback control strategies is assumed. This approach allows training of optimal feedback control strategies without explicit knowledge of the true solution.

FIGURE 6.1 shows the approach to training the neural controllers. Initial conditions in state space are generated randomly. A discretized version of the system is used along with the current parameterization (weight structure) of the neural controllers to close the loop via simulation. The states of both the controllers and system are generated at each time step and stored. At the end of the simulation the terminal cost is known.

FIGURE 6.1

NEURAL NET TRAJECTORY SHAPING VIA DIFFERENTIAL GAMES



The partials of this cost are fed back thru the system as shown in the Figure 6.1. A modified version of the backpropogation algorithm is used to adjust the neural controller weights. The backpropogation algorithm is fed for each time instance of the discretized system, the partial derivative of the terminal cost with respect to the state of the controller at that time instance. The backprogation algorithm then determines parameter adjustments (weight) for that time instance, along with the partial of the terminal cost with respect to the controllers input. The modification to the backpropogation algorithm comes from the fact the weight adjustments are accumulated over all time steps of the simulation. After feeding back the partial for all time steps the weights of the neural controllers are adjusted.

The simulation for the same initial conditions is repeated with the modified neural controllers. The adjustment simulation cycle is repeated until a change in the terminal cost falls below a predetermined threshold. This cycle is then repeated for new randomly generated initial states. The learning process is terminated when only minor adjustments are necessary for newly generated random initial conditions.

6.1 HARRIER CHASING AN F16 (ECCENTRIC AGGRESSIVE PEDESTRIAN)

The first differential game considered is that of an infinitely maneuverable pursuer against a faster evader with limited maneuverability. Asymptotically this can be viewed as a Harrier (infinitely maneuverable) against an F16 (faster). The game takes place in two dimensional planar space. This game can be used assuming altitude remains nearly constant during an encounter. Only the Harrier type aircraft is assumed to have aggressive capabilities, modeled by an eccentric circle terminal manifold. The game of degree for this type of terminal manifold is solved for the first time.

6.3.4. MODEL

The encounter takes place in two-dimensional realistic space (a fixed inertial coordinate system). The state space dimension is five. The model for the infinitely maneuverable aircraft (Harrier) is:

$$X_2^{\bullet} = w_2 \sin\Phi \qquad \text{(6.3.2)}$$
$$Y_2^{\bullet} = w_2 \cos\Phi$$

where

$w_2 \equiv$ speed of Harrier type aircraft

$\Phi \equiv$ control heading for Harrier type aircraft, measured from y-axis

The model for the faster but limited maneuverable aircraft (F16) is:

$$X_1^{\bullet} = w_1 \sin\theta$$
$$Y_1^{\bullet} = w_1 \cos\theta \qquad \text{(6.3.3)}$$
$$\theta^{\bullet} = w_1 \phi / R$$

where

$w_1 \equiv$ speed of F16 type aircraft

θ ≡ heading of F16 type aircraft measured from y-axis

R ≡ minimal turn radius of F16 type aircraft

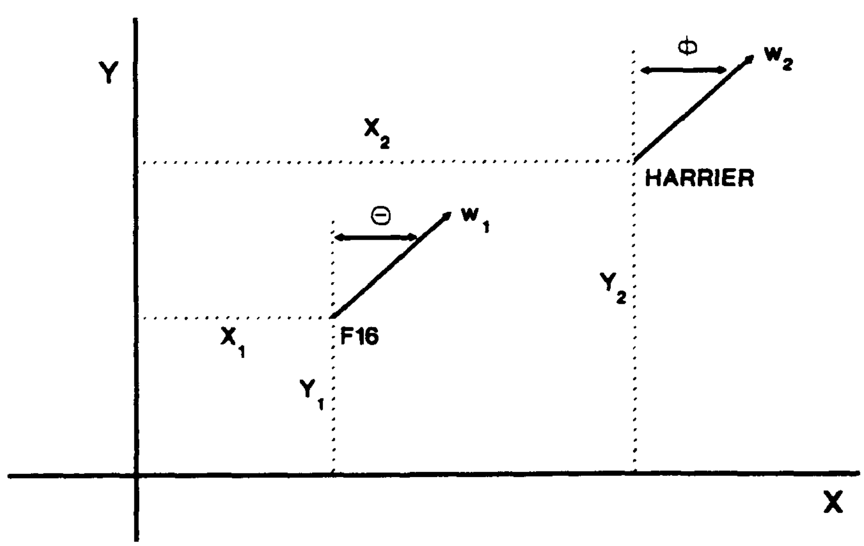
ϕ ≡ control for F16 type aircraft ($-1 < \phi < 1$)

note: -1 (1) _ maximum left (right) turn

A legitimate game requires $w_1 > w_2$. There would be no contest if the Harrier type aircraft was both infinitely maneuverable and faster. The two-dimensional realistic space model is shown in Figure 6.2.

The control strategies and target set are determined in a reduced state space. The dimension of the reduced state space is two. The new geometric coordinate system (a relative inertial coordinate system) is still two-dimensional. A right hand coordinate system with the new y-axis aligned with the F16 type aircraft velocity vector is used. The new coordinate system is relative to the F16 type aircraft.

FIGURE 6.2



Realistic Space

State equations for the new coordinate system are:

$$(6.3.3)\qquad \underline{X}^{\bullet} = -\frac{w_1}{R}\,\underline{Y}\phi + w_2\sin(\Phi-\theta)$$

$$\underline{Y}^{\bullet} = \frac{w_2}{R}\,\underline{X}\phi - w_1 + w_2\cos(\Phi-\theta).$$

Without loss of generality we introduce the new control for the Harrier, $\Phi^{*} = \Phi - \theta$. This leads to:

$$(6.3.4)\qquad \underline{X}^{\bullet} = -\frac{w_1}{R}\,\underline{Y}\phi + w_2\sin(\Phi^{*})$$

$$\underline{Y}^{\bullet} = \frac{w_2}{R}\,\underline{X}\phi - w_1 + w_2\cos(\Phi^{*}).$$

One last transformation is made to follow Davidovitz and Shinar (1985). A change of coordinates is introduced to normalize the system equations. The new coordinate transformation is:

$$(6.3.5)\qquad \underline{\underline{X}}(t) = \underline{X}(t\mathbf{R}/w_1)/\mathbf{R}$$

$$\underline{\underline{Y}}(t) = \underline{Y}(t\mathbf{R}/w_1)/\mathbf{R}.$$

This leads to the following normalized state equations:

$$(6.3.6)\qquad \underline{\underline{X}}^{\bullet} = -\underline{\underline{Y}}\phi + v\sin\Phi^{*}$$

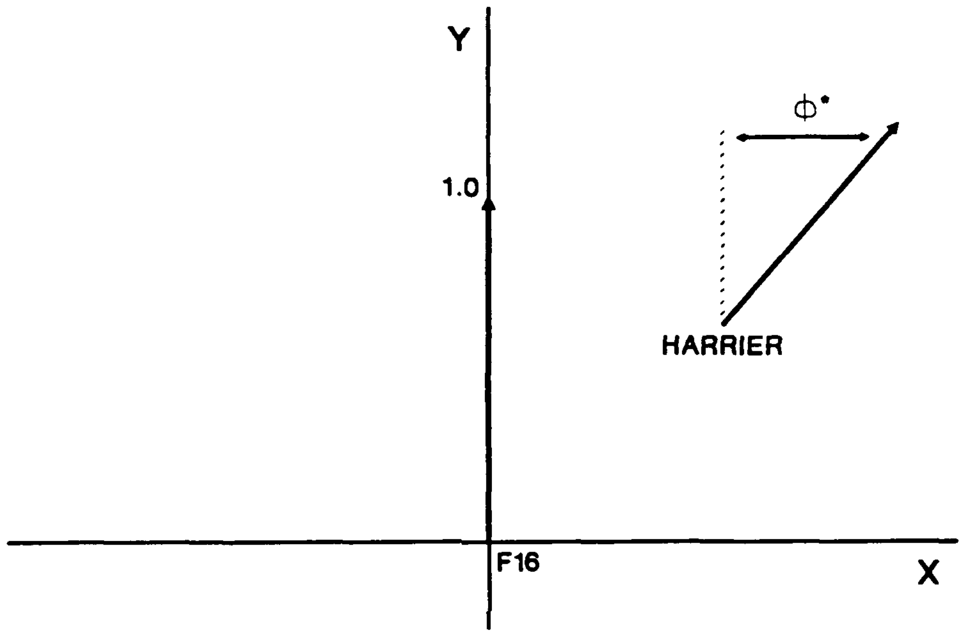
$$\underline{\underline{Y}}^{\bullet} = \underline{\underline{X}}\phi - 1 + v\cos\Phi^{*}$$

where:

$v \equiv$ the speed ratio w_2/w_1.

The normalized coordinate system relative to the F16 type aircraft is shown in Figure 6.3.

FIGURE 6.3

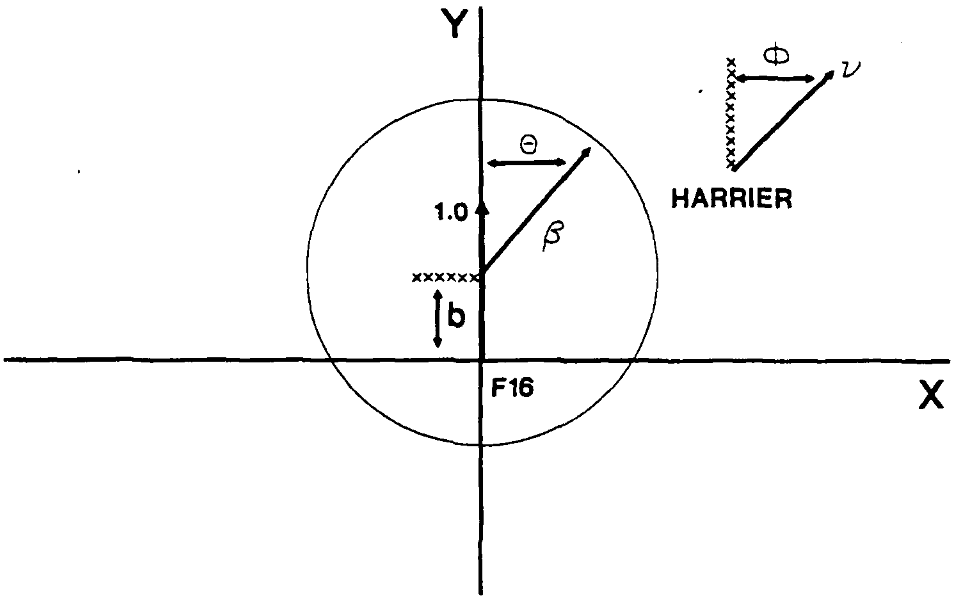


COORDINATE SYSTEM RELATIVE TO F16 TYPE AIRCRAFT

6.3.5 STRATEGY

Only the Harrier type aircraft has aggressive capabilities. The terminal manifold is an eccentric circle centered at the F16 in the reduced state space of equations (6.3.6). The eccentricity is positive along the Y-axis. The model for the encounter is shown in Figure 6.4. The goal of the Harrier type aircraft is to penetrate the eccentric "kill" circle centered at the F16 type aircraft.

FIGURE **6.4**



Harrier Type Aggressor

The model variables are:

b ≡ Eccentricity of Harrier type kill circle

β ≡ Radius of Harrier type kill circle

v ≡ Velocity ratio: $v < 1$ (Harrier type aircraft slower than F16 type aircraft)

θ ≡ Parameter of Harrier type kill circle (terminal manifold)

X,Y ≡ Cartesian coordinates in frame of the F16 type aircraft with F16 type velocity aligned with y-axis

The reduced state equations 6.3.7 are repeated here for clarity:

(6.3.8)
$$X^{\bullet} = -Y\phi + v\sin\Phi$$
$$Y^{\bullet} = X\phi - 1 + v\cos\Phi$$

where:

Φ ≡ Harrier type aircraft control

ϕ ≡ F16 type aircraft control: $|\phi| < 1$.

The solution must be found in retro-time. We introduce the retro time variable t as follows:

$$\tau = t_f - t$$

where:

t ≡ forward time

t_f ≡ final time (when kill circle is penetrated)

The final conditions are:

$$X_f = \beta\sin(\theta) \qquad \textbf{(6.3.8)}$$
$$Y_f = \beta\cos(\theta) + b$$

Section 2.2.2 page 32 showed that the terminal manifold is partitioned into the usable part (UP), non-usable part (NUP), and boundary of the usable part (BUP). The BUP and UP regions are:

$$\text{BUP} \equiv \left[(X_f, Y_f) : \ \theta_{u\beta} = \cos^{-1}\left[\frac{\nu + b(1+b^2-\nu^2)^{1/2}}{(1+b^2)} \right] \right] \qquad \textbf{(6.3.9)}$$

$$\text{UP} \equiv ((X_f, Y_f) : \ |\theta| < \theta_{u\beta})$$

The next step is to determine the capture region and barriers defined in section 2.2 page 32 (the game of kind). Clearly the barriers enclose the capture region as a subset of the game space because the Harrier type aircraft is slower than the F16 type aircraft. When the velocity vectors of both the Harrier and F16 type aircraft are aligned, it is impossible for the Harrier to close distance. The barrier equations for the right half plane (RHP) (SHINAR 19851) are:

$$X(\tau) = -b\sin\tau + \cos\tau - 1 + (\nu\tau + \beta)\sin(\theta_{u\beta} - \tau)$$

(6.3.11)

$$Y(\tau) = b\cos\tau + \sin\tau + (\nu\tau + \beta)\cos(\theta_{u\beta} - \tau).$$

Paths must be determined for all points lying in the capture region by the game of degree (section 2.1). A cost function is assigned so that all the possible paths terminating on the kill circle can be compared. The cost function used by the Harrier is the time to the kill circle (terminal manifold). This cost function is:

(6.3.12) $$\Gamma(X,Y) = \int dt$$

Isaacs' main equation (2.1.32) is:

(6.3.13)

$$\min_{\Phi}\max_{\phi}[\phi(-Y\Gamma_X+X\Gamma_Y)+\nu(\Gamma_X{}^2+\Gamma_Y{}^2)^{1/2}\cos(\Phi\text{-}\tan\left[\frac{\Gamma_X}{\Gamma_Y}\right]) - \Gamma_Y + 1] = 0$$

This leads to the optimal controls for the right half plane (RHP):

$$\Phi = \pi + \tan^{-1}\left[\frac{\Gamma_X}{\Gamma_Y}\right]$$

(6.3.14)

$$\phi = \text{sign}\{-Y\Gamma_X + X\Gamma_Y\} = \text{sign}\{S\}$$

The terminal conditions are:

$$X_f = \beta\sin(\theta)$$

$$Y_f = \beta\cos(\theta) + b$$

(6.3.15) $$\Gamma(\theta) = 0$$

$$S = -b\Gamma_X$$

$$0 = \Gamma_X \cos\theta - \Gamma_Y \sin\theta \quad \text{(Tranversality)}$$

These terminal conditions leads to the following terminal controls and costates for the RHP :

$$\Phi = \pi + \theta$$

$$\phi = -1$$

$$\Gamma_X = \left[\frac{\sin\theta}{v + \cos\theta - b\sin\theta} \right]$$

$$\Gamma_Y = \left[\frac{\cos\theta}{v + \cos\theta - b\sin\theta} \right]$$

The retro state and costate equations are:

(6.3.16)

$$\Gamma_X^{\circ} = \Gamma_Y \phi$$

$$\Gamma_Y^{\circ} = -\Gamma_X \phi$$

$$X^{\circ} = Y\phi - v\sin\Phi$$

$$Y^{\circ} = -X\phi + 1 - v\cos\Phi$$

Integration of these retro equations with the terminal conditions, yield for the RHP:

(6.3.17)

$$\Phi = \pi + \theta - \tau$$

$$\phi = -1$$

$$X(\tau) = -b\sin\tau + \cos\tau - 1 + (v\tau + \beta)\sin(\theta - \tau)$$

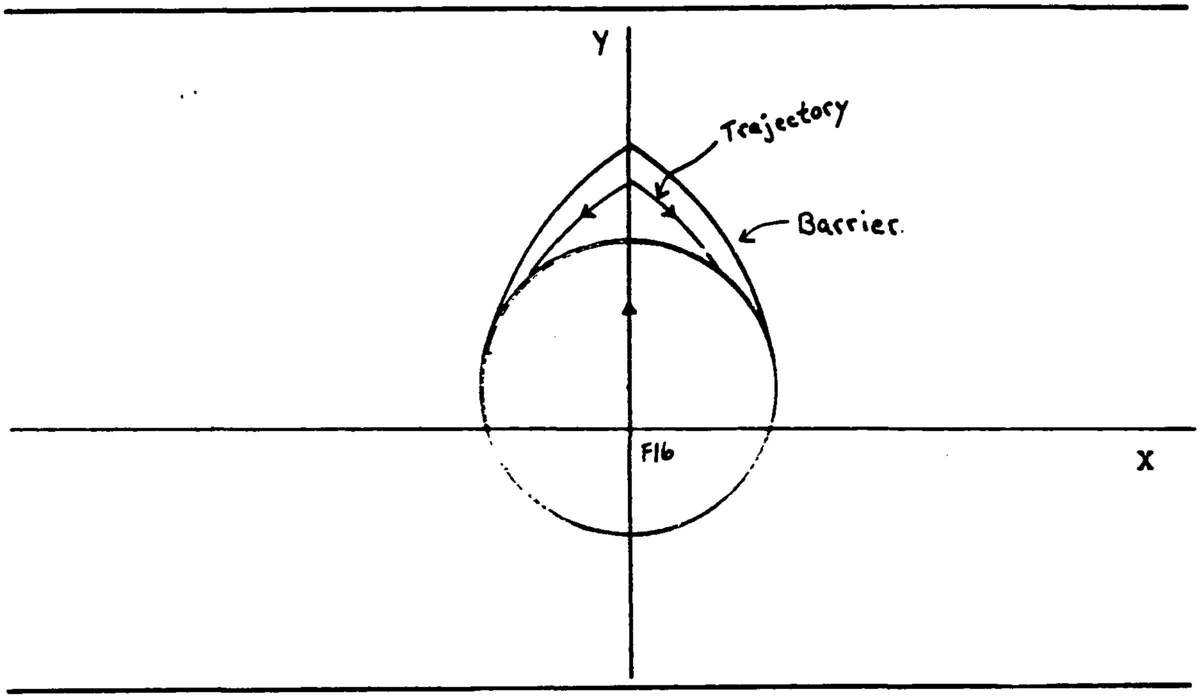
$$Y(\tau) = b\cos\tau + \sin\tau + (v\tau + \beta)\cos(\theta - \tau)$$

The optimal controls are found by inverting the last two equations of (6.3.16) to determine θ and τ. Unfortunately the equations are non-linear and the inversion must be done numerically.

An important point to note is that the barrier is a smooth imbedding in the one parameter (θ) family of paths. This is important when a differential equation is approximated by a difference equation. This smoothness property means that the strategies played cannot change drastically during an integration update period for the approximating difference equation. This will not be true when the F16 type aircraft is considered the aggressor.

One final note is the y-axis serves as a dispersal surface. For any point on the y-axis there are two optimal paths, one to the right and one to the left. These paths are mirror images of each other, due to the symmetry of the problem. The Harrier wishes to stay on the y-axis, but the optimal move for the F16 is to force the trajectory off the y-axis. The barrier and optimal trajectories are shown in the figure below.

FIGURE 6.5
Barriers and Optimal Trajectories For The Capture Region Harrier type Aircraft
The Aggressor

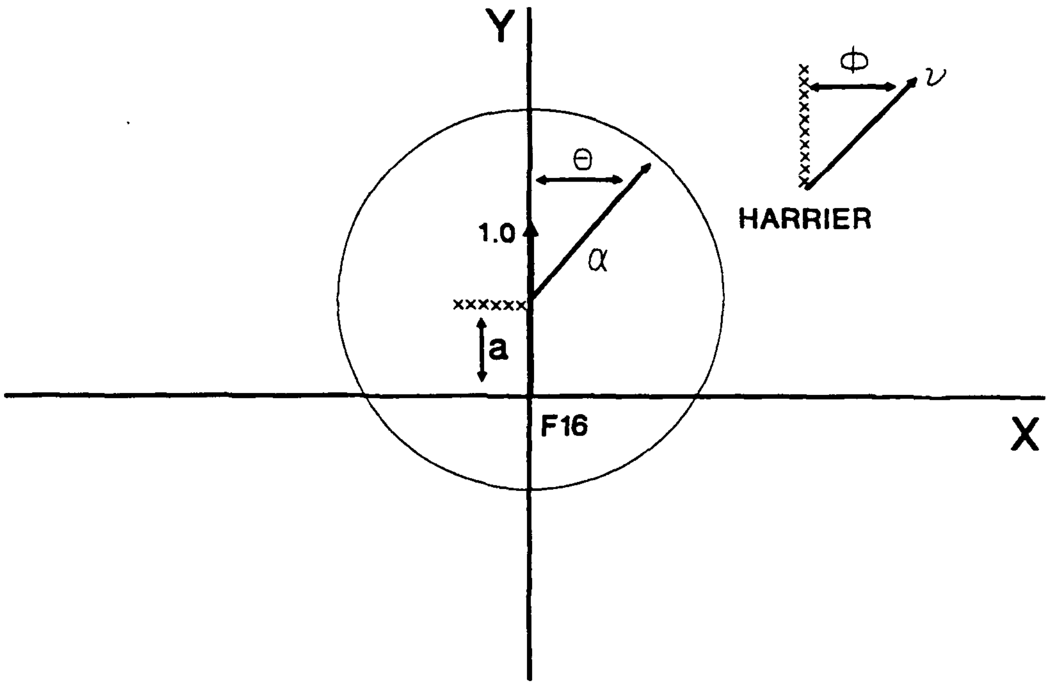


6.2. F16 CHASING A HARRIER (ECCENTRIC HOMICIDAL CHAUFFEUR)

The second differential game considered is that of a faster pursuer against a slower but infinitely maneuverable evader. The role of pursuer and evader have been exchanged from that of section 6.3 (Harrier Chasing An F16). The F16 type aircraft is now assumed to be the aggressor. This game also takes place in two dimensional planar space. Only the F16 type aircraft is now assumed to have aggressive capabilities, modeled by an eccentric circle terminal manifold. The game of degree for this type of terminal manifold is solved for the first time.

The fixed inertial model is the same as in section 6.3.4 page 135. The F16 type aircraft now has the aggressive capabilities. The terminal manifold is an eccentric "kill" circle centered at the F16 in the reduced state space (shown in Figure 6.3). The eccentricity is positive along the Y-axis. The goal of the F16 type aircraft is to force the Harrier type aircraft to penetrate the terminal manifold. The relative coordinate frame model is shown in Figure 6.6.

FIGURE 6.6



F16 Type Aggressor

The model variables are:

a ≡ Eccentricity of F16 type kill circle (terminal manifold)

α ≡ Radius of F16 type kill circle (terminal manifold)

ν ≡ Velocity ratio: $\nu < 1$ (Harrier type aircraft slower than F16 type aircraft)

θ ≡ Parameter of F16 type kill circle (terminal manifold)

X,Y ≡ Cartesian coordinates in frame of the F16 type aircraft with F16 type velocity aligned with y-axis

The state equations and controls are the same as in the case that the Harrier was the aggressor (equations (6.3.8)). The solution again must be found in retro time (τ). The terminal conditions are:

$$
\text{(6.3.17)} \qquad
\begin{aligned}
X_f &= \alpha\sin\theta \\
Y_f &= \alpha\cos\theta + a
\end{aligned}
$$

The usable part (UP) and the boundary of the usable part (BUP) of the terminal manifold (kill circle) are:

$$
\text{(6.3.18)} \qquad
\begin{aligned}
\text{BUP} &\equiv \left[(X_f, Y_f) \colon \theta_{u\alpha} = \cos^{-1}\left[\frac{-v + a(1+a^2-v^2)^{1/2}}{(1+a^2)} \right] \right] \\
\text{UP} &\equiv ((X_f, Y_f) \colon |\theta| < \theta_{u\alpha})
\end{aligned}
$$

Two initial assumptions will be used here. The first is that the whole space is the capture region. This implies that the barriers (if they exist) do not meet (are open). The mathematical condition for open barriers from Shinar (1985) is:

$$
\text{(6.3.19)} \qquad \alpha > (1 + a^2 - v^2)^{1/2} + v\sin^{-1}(\cos\theta_{u\alpha}) - 1
$$

The second condition is that the barriers meet the "kill" circle tangentially. This mathematical condition from Shinar (1985) is:

$$
\text{(6.3.20)} \qquad \alpha^2 > (1 + a^2 - v^2)
$$

The barrier equations for the right half plane (RHP) from Shinar (1985) are:

(6.3.21)
$$X(\tau) = a\sin\tau - \cos\tau + 1 + (\alpha - \nu\tau)\sin(\tau + \theta_{u\alpha}),$$
$$Y(\tau) = a\cos\tau + \sin\tau + (\alpha - \nu\tau)\cos(\tau + \theta_{u\alpha}),$$

where $\tau \leq 2\Pi - \cos^{-1}(\nu) - \theta_{u\alpha}$[1]

A cost function is again needed to choose a trajectory leading to the terminal manifold. The cost function used by the Harrier is the time to the kill circle (terminal manifold), but in this case it is the time till the Harrier type aircraft reaches the F16 kill circle. The cost function is:

$$\Gamma(X,Y) = \int dt$$

Isaacs' main equation[2] (2.1.32) is :

(6.3.22)

$$\max_{\Phi} \min_{\phi} [\phi(-Y\Gamma_X+X\Gamma_Y)+\nu(\Gamma_X{}^2+\Gamma_Y{}^2)^{1/2}\cos(\Phi\text{-}\tan^{-1}\left[\frac{\Gamma_X}{\Gamma_Y}\right])-\Gamma_Y+1] = 0$$

---

[1] The barrier termination time.

[2] The role of minimizer and maximizer have now been switched form the case where the Harrier type aircraft is the aggressor.

This leads to the optimal controls for the right half plane (RHP):

$$\Phi = \tan^{-1}\left[\frac{\Gamma_X}{\Gamma_Y}\right]$$

(6.3.23)

$$\phi = -\text{sign}\{-Y\Gamma_X + X\Gamma_Y\} = -\text{sign}\{S\}$$

The games space must be decomposed according to section 2.1.6 page 13. Each of the cases discussed below corresponds to a region E_{ij} in which the optimal controls are continuous. The singular surfaces that function as borders (controls are discontinuous) for the different cases correspond to the M_{ij}. Equation (2.1.47) is used to generate the initial conditions on the singular surfaces.

The first case to be considered is the Harrier type aircraft below the barrier. This corresponds to the classical situation in which the pursuer (F16) must perform a swerve maneuver. A universal surface exists along the y-axis both above and below the kill circle (terminal manifold). The y-axis below the kill circle is a universal surface because by symmetry there are trajectories entering from each half plane. These trajectories upon entry continue along the universal surface[3]. This corresponds to a tail chase with the Harrier type aircraft chasing the F16 type.

[3]A further discussion of universal surfaces and their implication can be found in Issaacs (2).

The universal surface below the y-axis will be used as an intermediate terminal manifold to generate the swerve maneuver. The terminal conditions are with parameter s, ($s < a - \alpha < 0$):

$$X_f = 0$$

(6.3.25) $$Y_f = s$$

$$\Gamma(s) = \frac{(s - (\alpha - a))}{(1 - \nu)} + \text{constant (tranversality)}$$

$$\max_{\Phi} \min_{\phi} [\phi(-s\Gamma_X) + \nu(\Gamma_X{}^2 + \Gamma_Y{}^2)^{1/2}\cos(\Phi - \tan^{-1}\left[\frac{\Gamma_X}{\Gamma_Y}\right]) - \Gamma_Y + 1] = 0$$

These terminal conditions lead to the following terminal controls and costates for the RHP:

$$\Gamma_Y = (1 - \nu)^{-1}$$

$$\Gamma_X = 0$$

(6.3.26) $$\Phi_F = 0$$

$$\phi_F = -1$$

$$S_F = 0.$$

The retro[4] state and costate equations are:

$$\Gamma_X{}^\circ = \Gamma_Y\, \phi$$

$$\Gamma_Y{}^\circ = \Gamma_X\, \phi$$

(6.3.27) $$X^\circ = Y\phi - \nu\sin\Phi$$

$$Y^\circ = -X\phi + 1 - \nu\cos\Phi$$

$$S^\circ = -\Gamma_X$$

---

[4]Retro time τ is now the time to the universal surface, not the time to the kill circle.

Integration of these retro equations with the terminal conditions yield for the RHP:

$$\begin{aligned}
&\Phi(\tau) = -\tau \\
&\phi(\tau) = -1 \\
&X(\tau) = (\nu\tau - s)\sin\tau + \cos\tau - 1 \\
&Y(\tau) = -(\nu\tau - s)\cos\tau + \sin\tau
\end{aligned} \tag{6.3.27}$$

Solving for s and τ[5]:

$$\begin{aligned}
&s = ((X + 1)^2 + Y^2 - 1)^{1/2} + \nu\tau \\
&\tau = \tan^{-1}(Y/(X+1)) + \tan^{-1}(((X+1)^2 + Y^2 - 1)^{1/2})
\end{aligned} \tag{6.3.28}$$

It should also be noted at this point that extension of the barrier equations from the barrier termination point at $\tau = 2\Pi - \cos^{-1}(\nu) - \theta_{u\alpha}$ till the barrier extension intersects the negative Y-axis, is a singular surface. Once a trajectory starting below the barrier reaches this surface the swerve portion of the maneuver can be started. At this point the pursuer (F16) begins to turn to start chasing the evader (Harrier), while the Harrier also starts turning to begin evading.

There are two cases to be considered for the swerve portion of the maneuver, trajectories that terminate directly on the terminal manifold, and trajectories that first terminate on a universal surface consisting of the Y-axis above the terminal manifold and then continuing on the Y-axis to the terminal manifold.

---

[5]The dependency of τ only on the states X,Y allows the use of state feedback for Φ.

We now consider the case where trajectories terminate directly on the terminal manifold. The main equation and optimal control solution equations are:

(6.3.30)

$$\max_{\Phi} \min_{\phi}[\phi(-Y\Gamma_X+X\Gamma_Y)+\nu(\Gamma_X^2+\Gamma_Y^2)^{1/2}\cos(\Phi-\tan^{-1}\left[\frac{\Gamma_X}{\Gamma_Y}\right])-\Gamma_Y+1] = 0$$

(6.3.31)

$$\Phi = \tan^{-1}\left[\frac{\Gamma_X}{\Gamma_Y}\right]$$

$$\phi = -\mathrm{sign}\{-Y\Gamma_X+X\Gamma_Y\} = -\mathrm{sign}\{S\}$$

The terminal conditions with parameter θ ($0 \le \theta \le \theta_{u\alpha}$) are:

(6.3.32)

$$X_f = \alpha\sin(\theta)$$

$$Y_f = \alpha\cos(\theta) + a$$

$$\Gamma(\theta) = 0$$

$$\Gamma_X\cos(\theta) - \Gamma_Y\sin(\theta) = 0$$

$$S = -a\Gamma_X$$

These terminal conditions lead to the following terminal controls and costates for the RHP:

(6.3.33)

$$\Gamma_X = \sin(\theta)$$

$$\Gamma_Y = \cos(\theta)$$

$$\Phi = \theta$$

$$\phi = 1$$

The retro state and costate equations are:

(6.3.34)

$$\Gamma_X^\circ = -\Gamma_Y$$

$$\Gamma_Y^\circ = \Gamma_X$$

$$X^\circ = -Y + \nu\sin\tau$$

$$Y^\circ = X + 1 - \nu\cos\tau$$

$$S^{\circ} = -\Gamma_X$$

Integration of these retro equations with the terminal conditions, yield for the RHP:

(6.3.35)

$$\Gamma_X = \sin(\tau + \theta)$$
$$\Gamma_Y = \cos(\tau + \theta)$$
$$S = -\cos(\tau + \theta) - a + \cos(\theta)$$
$$\Phi = \theta + \tau$$
$$\phi = 1$$
$$X(\tau) = (\alpha - \nu\tau)\sin(\theta+\tau) + a\sin\tau - \cos\tau + 1$$
$$Y(\tau) = (\alpha - \nu\tau)\cos(\theta+\tau) + a\cos\tau + \sin\tau$$

Once again to determine the optimal controls we must determine τ and θ from the equations for $X(\tau)$ and $Y(\tau)$ above. The problem is that a closed form solution does not exist and numerical methods must be relied on.

The last case to be considered is for those trajectories that terminate on the Y-axis universal surface above the terminal manifold. The encounter then ends with a classical tail chase of the pursuer (F16) chasing the evader (Harrier). The main equation and optimal control solution equations remain the same as in the case of termination directly on the terminal manifold. The difference is that the Y-axis above the terminal manifold now serves as an intermediate terminal manifold for the trajectories. This means that we are now looking for trajectories that terminate on the following surface parameterized by σ ($\alpha + a \leq \sigma$):

(6.3.36)

$$X_f(\sigma) = 0$$
$$Y_f(\sigma) = \sigma$$

The other terminal condition is:

$$S^{\circ} = -\Gamma_X$$

Integration of these retro equations with the terminal conditions, yield for the RHP:

(6.3.34)
$$\Gamma_X = \sin(\tau + \theta)$$
$$\Gamma_Y = \cos(\tau + \theta)$$
$$S = -\cos(\tau + \theta) - a + \cos(\theta)$$
$$\Phi = \theta + \tau$$
$$\phi = 1$$
$$X(\tau) = (\alpha - \nu\tau)\sin(\theta+\tau) + a\sin\tau - \cos\tau + 1$$
$$Y(\tau) = (\alpha - \nu\tau)\cos(\theta+\tau) + a\cos\tau + \sin\tau$$

Once again to determine the optimal controls we must determine τ and θ from the equations for $X(\tau)$ and $Y(\tau)$ above. The problem is that a closed form solution does not exist and numerical methods must be relied on.

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(6.3.35)
$$X_f(\sigma) = 0$$
$$Y_f(\sigma) = \sigma$$

The other terminal condition is:

(6.3.37) $\Gamma(\sigma) = (\sigma - (\alpha + a))/(1 - \nu)$

This leads to the following terminal controls and costates for the RHP:

(6.3.38)

$$\Gamma_Y = 1/(1 - \nu)$$

$$\Phi = 0$$

$$\phi = 1$$

$$\Gamma_X = 0 \text{ (continuity on US)}$$

The retro state and costate equations are:

(6.3.39)

$$\Gamma_X^\circ = \Gamma_Y \phi$$

$$\Gamma_Y^\circ = -\Gamma_X \phi$$

$$S^\circ = -\Gamma_X$$

$$X^\circ = -Y + \nu \sin\tau$$

$$Y^\circ = X + 1 - \nu \cos\tau$$

Integration of these retro equations with the terminal conditions, yield for the RHP:

(6.3.40)

$$\Gamma_X = \frac{\sin(\tau)}{1 - \nu}$$

$$\Gamma_Y = \frac{\cos(\tau)}{1 - \nu}$$

$$S = (\cos(\tau) - 1)/(1 - \nu)$$

$$\Phi = \tau$$

$$\phi = 1$$

$$X(\tau) = \sigma \sin\tau - \cos\tau \quad - \nu\tau \sin\tau$$

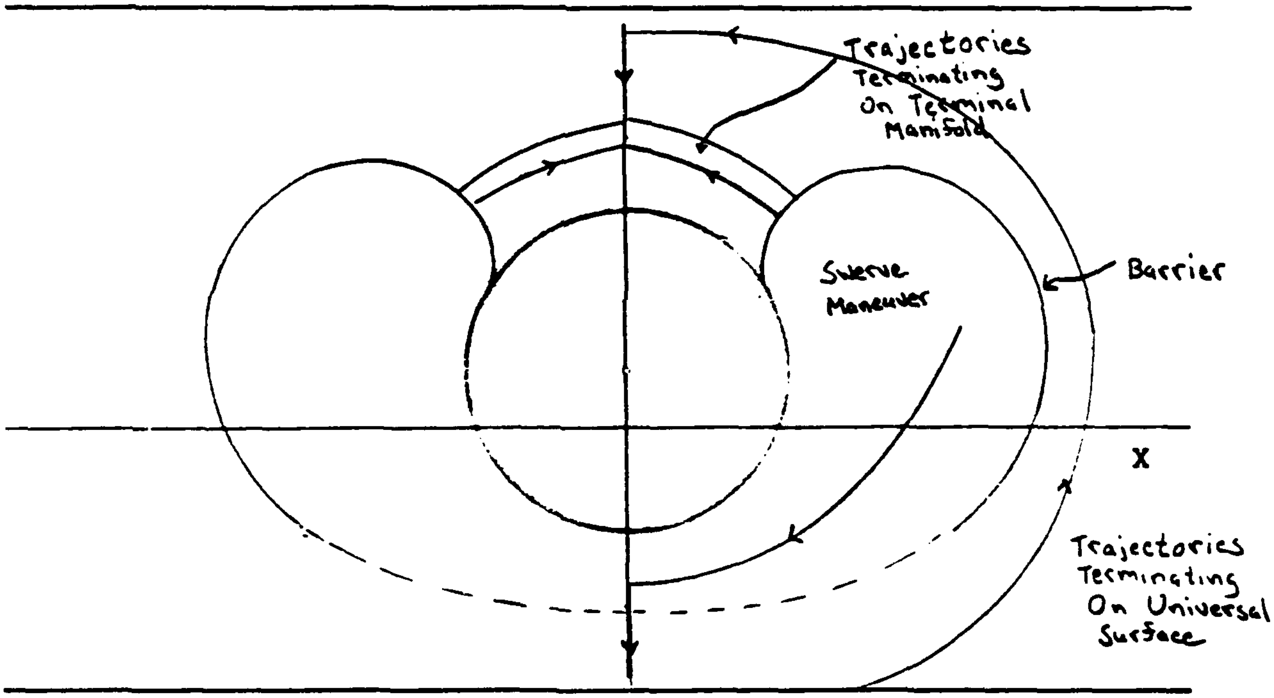
$$Y(\tau) = \sigma \cos\tau + \sin\tau - \nu\tau \cos\tau$$

The closed loop optimal control can be found directly from the last two equations as:

$$\tau = \tan^{-1}((X-1)/Y) + \tan^{-1}(((X-1)^2 + Y^2 - 1)^{-1/2}) \qquad (6.3.41)$$

The barrier and optimal trajectories are shown in the Figure 6.7.

FIGURE 6.7
Barriers and Optimal Trajectories For The Capture Region
F16 type Aircraft The Aggressor



7. CONTRIBUTIONS, SUMMARY AND FUTURE RESEARCH

We have extensively explored the use of low order differential games in flight and fire control via artificial intelligence methodologies. Differential game technology currently, and for the foreseeable future, will only be able to generate solutions to low dimensional models. Unfortunately, these low dimensional models by themselves are not usable for actual air-combat. This dissertation has developed methodologies for dealing with extremely complex dynamical systems (such as air-combat) with only a partial understanding of the true system nature.

We have given an appropriate derivation of the necessary conditions for the differential game of kind and game of degree. Concentration has been on differential game theory actually applicable to air-combat. A simplified development of the game of degree necessary conditions combines Berkovitz's methodology with Isaacs' *theorem* (that all games are equivalent to an autonomous game with terminal payoff). The consideration of only autonomous feedback strategies when dealing with autonomous systems has extended Berkovitz's results, which depended always on using the family of time varying feedback strategies in the variation.

We have also explored the nature of complex systems in which the form and order changes. This exploration has reviewed and extended Mesarovic's definition of a general system. Traditional system theory falls under Mesarovic's definition in which the relations and components remain fixed. Complex systems such as air-combat do not fall in this category. The addition (reproduction) and subtraction (attrition) of components modifies the system relations. Because of this we have extended Mesarovic's definition to handle self modifying systems and have termed these systems Semantic Systems.

We have shown that Semantic Systems naturally relate to the Semantic Control Paradigm. Future research is still open concerning conditions under which key Semantic System Components are survivable and reliable. Survivability relates to the conditions under which a component can prevent its own removal from the system (semantic order). Reliability relates to the conditions under which a component(s) can force a particular semantic state. Survivability and reliability are the Semantic System extensions of reachability and controllability.

In addition, we have also created a knowledge base of differential. Derived for the first time were the game of degree solutions of an eccentric terminal manifold for a Harrier evader and F16 pursuer. These game solutions are currently in our air-combat simulator derived under our new Frame Based Simulation methodology for Semantic Systems. Current involvement includes research to increase the knowledge base of game solutions via artificial neural nets. The approach is to train individual Neural Nets to be the optimal feedback controllers for the pursuer and evader. The only information assumed to be available is the initial state of the system, and the final state of the system via simulation. This approach assumes no knowledge of the true optimal trajectories or feedback control strategies. This approach allows training of optimal feedback control strategies without explicit knowledge of the true solution.

We have explored artificial intelligence methods to determine the most appropriate differential game from the knowledge base. Development has also included a probabilistic missile effectiveness model, and a methodology to get around the zero-one terminal manifold inherent in differential games. The zero-one terminal manifold means a missile has zero probability of destroying an adversary until it reaches a surface at which point the missile has 100% probability of destroying the adversary.

The general approach taken in this work has been to combine different technologies that by themselves are incapable of solving the problem at hand. Hybridization techniques for future integrated system methodologies will solve problems that are currently beyond the scope of pure solutions. Methodologies in this work have application to problems currently deemed so difficult that no appropriate mathematical model can be derived, nor can be found any appropriate classical analytical methods.

We have outlined three AI methodologies for splicing together the differential games; rule base expert systems, artificial neural nets, and the Analytical Hierarchy Process (AHP). The rule based expert system appears to be the easiest methodology for immediate implementation, but requires a deep understanding of the game in the knowledge base and is brittle to incrementing the knowledge base with more games. The artificial neural net implementation offers very high performance via training, but training sets would be very difficult go generate. The AHP appears to offer the best compromise. It requires only local understanding via the pairwise comparison and its hierarchic structure. The AHP also would easily incorporate incrementing the knowledge base. The question of appropriate criteria for the hierarchies of the AHP are still open to research. Training methodologies for the artificial neural net approach also are still open. Lastly a comparison via simulation of the performance of the three methodologies is still forthcoming.

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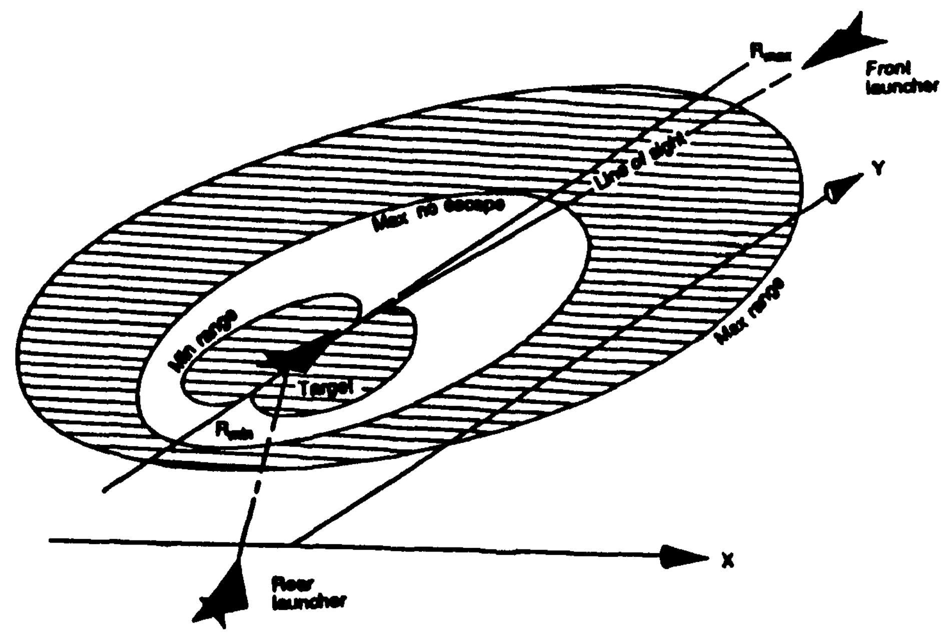
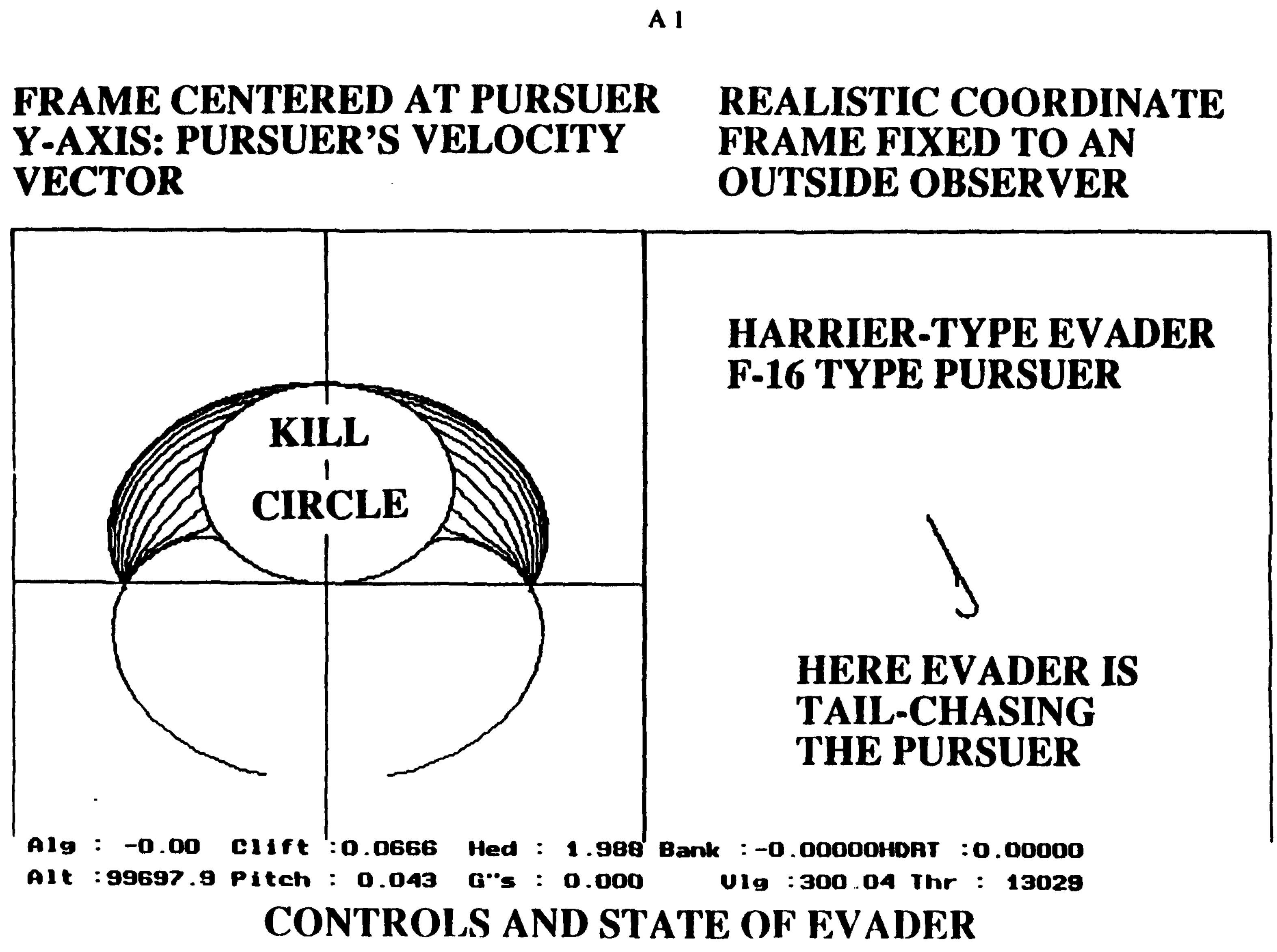
10. APPENDIX A: SIMULATOR RESULTS

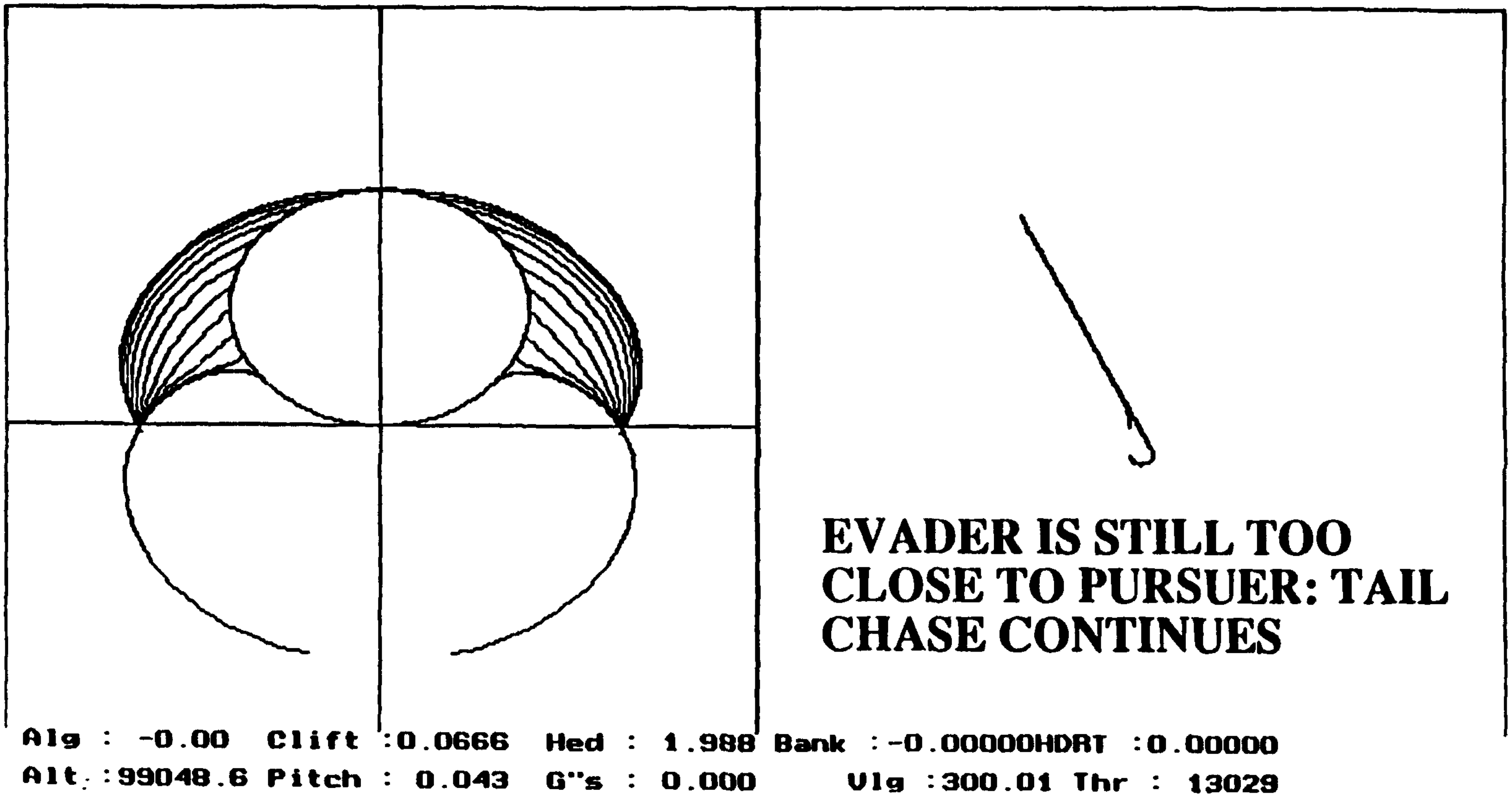
This section contains two example runs from the simulator. The first example shows the result of choosing the wrong strategy. The pursuer chooses a line of sight strategy and tries to turn too quickly and is unable to capture the evader. The second example shows the result of a proper choice of strategies by both players.

Missile Launch Envelope

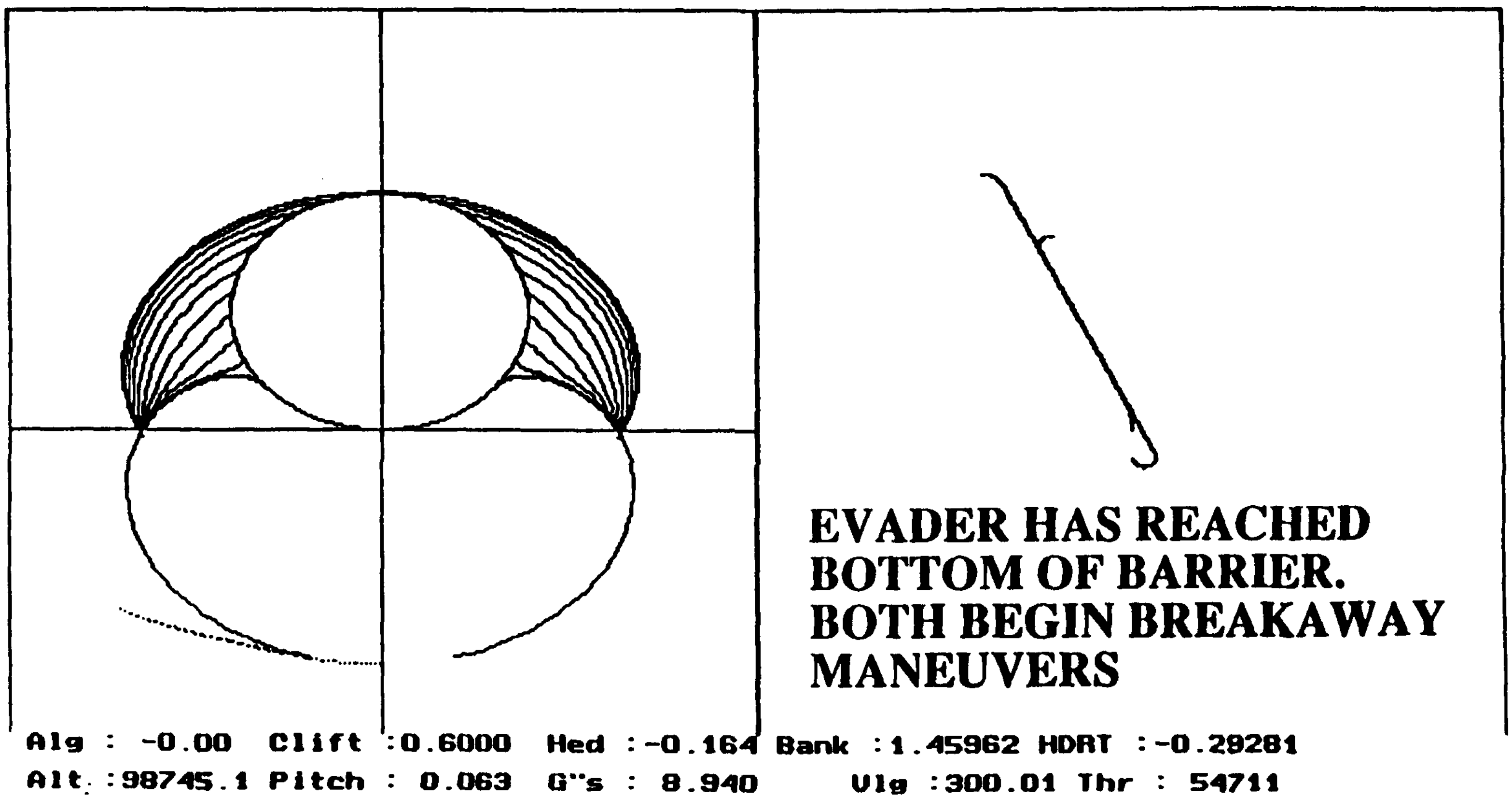



A I
FRAME CENTERED AT PURSUER
Y-AXIS: PURSUER'S VELOCITY
VECTOR
REALISTIC COORDINATE
FRAME FIXED TO AN
OUTSIDE OBSERVER
KILL
CIRCLE
HARRIER-TYPE EVADER
F-16 TYPE PURSUER
HERE EVADER IS
TAIL-CHASING
THE PURSUER
Alg : -0.00 Clift :0.0666 Hed : 1.988 Bank :-0.00000HDRT :0.00000
Alt :99697.9 Pitch : 0.043 G"s : 0.000 Vlg :300.04 Thr : 13029
CONTROLS AND STATE OF EVADER

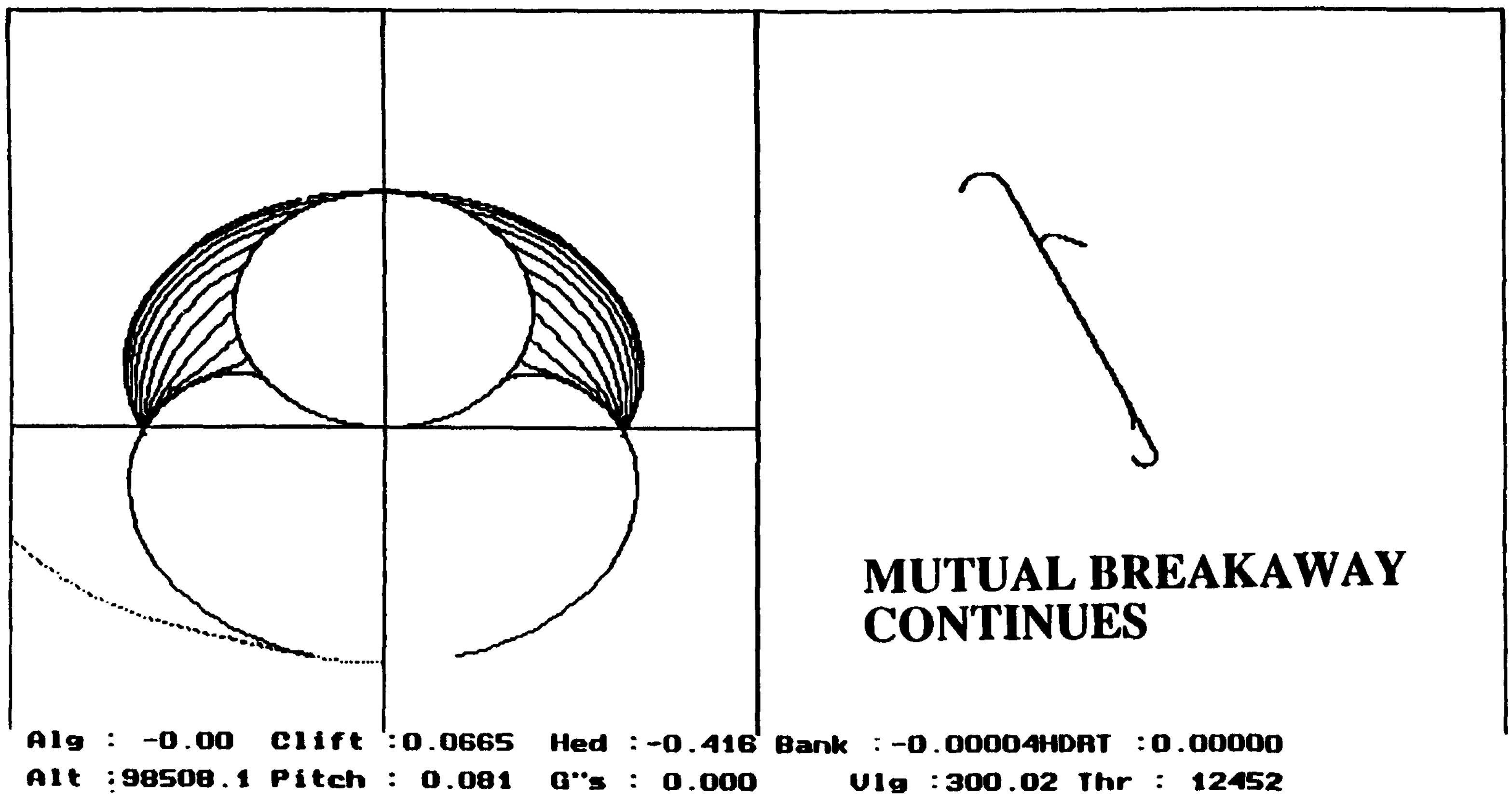
A 2


EVADER IS STILL TOO CLOSE TO PURSUER: TAIL CHASE CONTINUES
Alg : -0.00 Clift :0.0666 Hed : 1.988 Bank :-0.00000HDRT :0.00000
Alt :99048.6 Pitch : 0.043 G"s : 0.000 Vlg :300.01 Thr : 13029

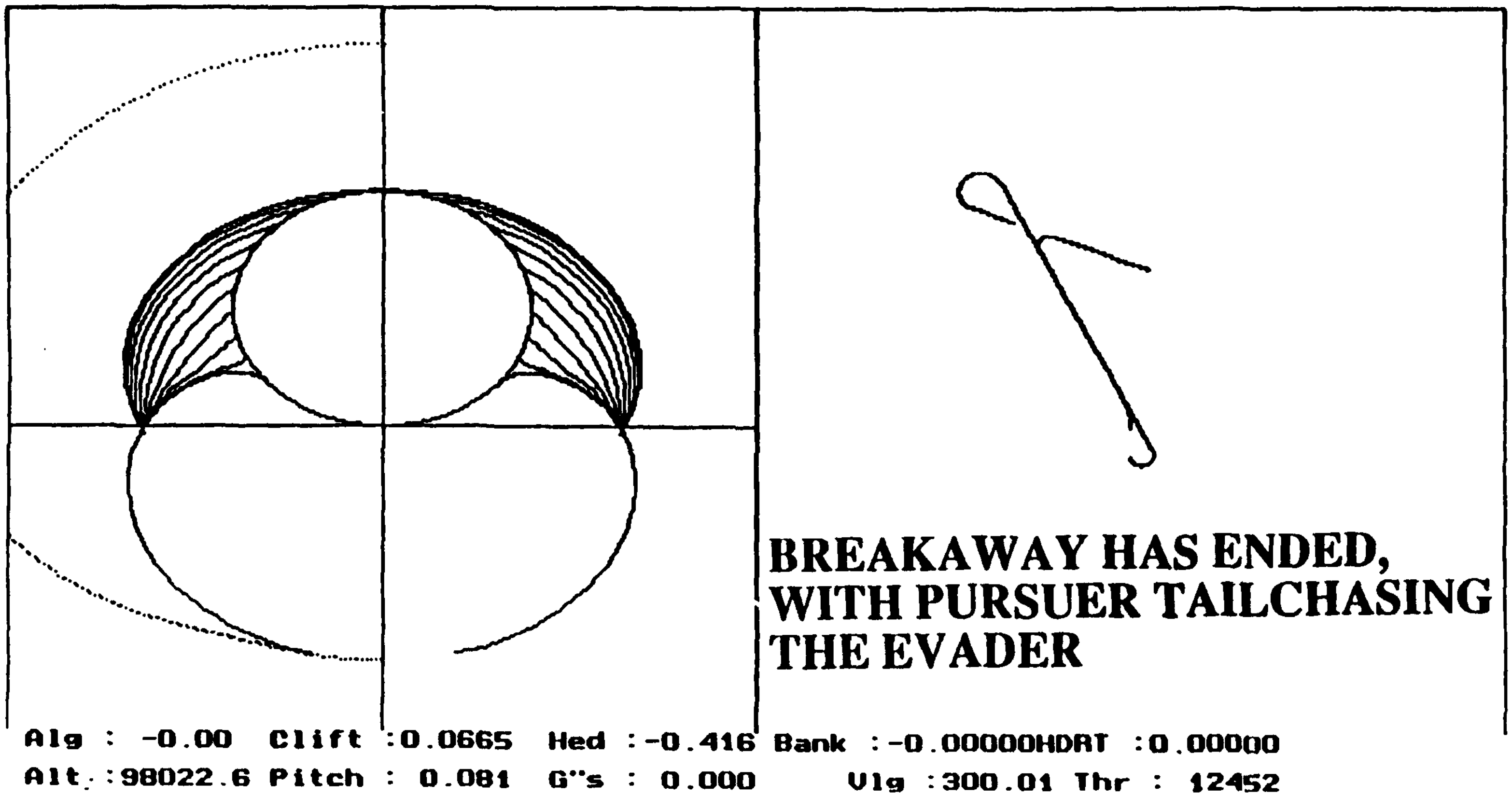
A 3

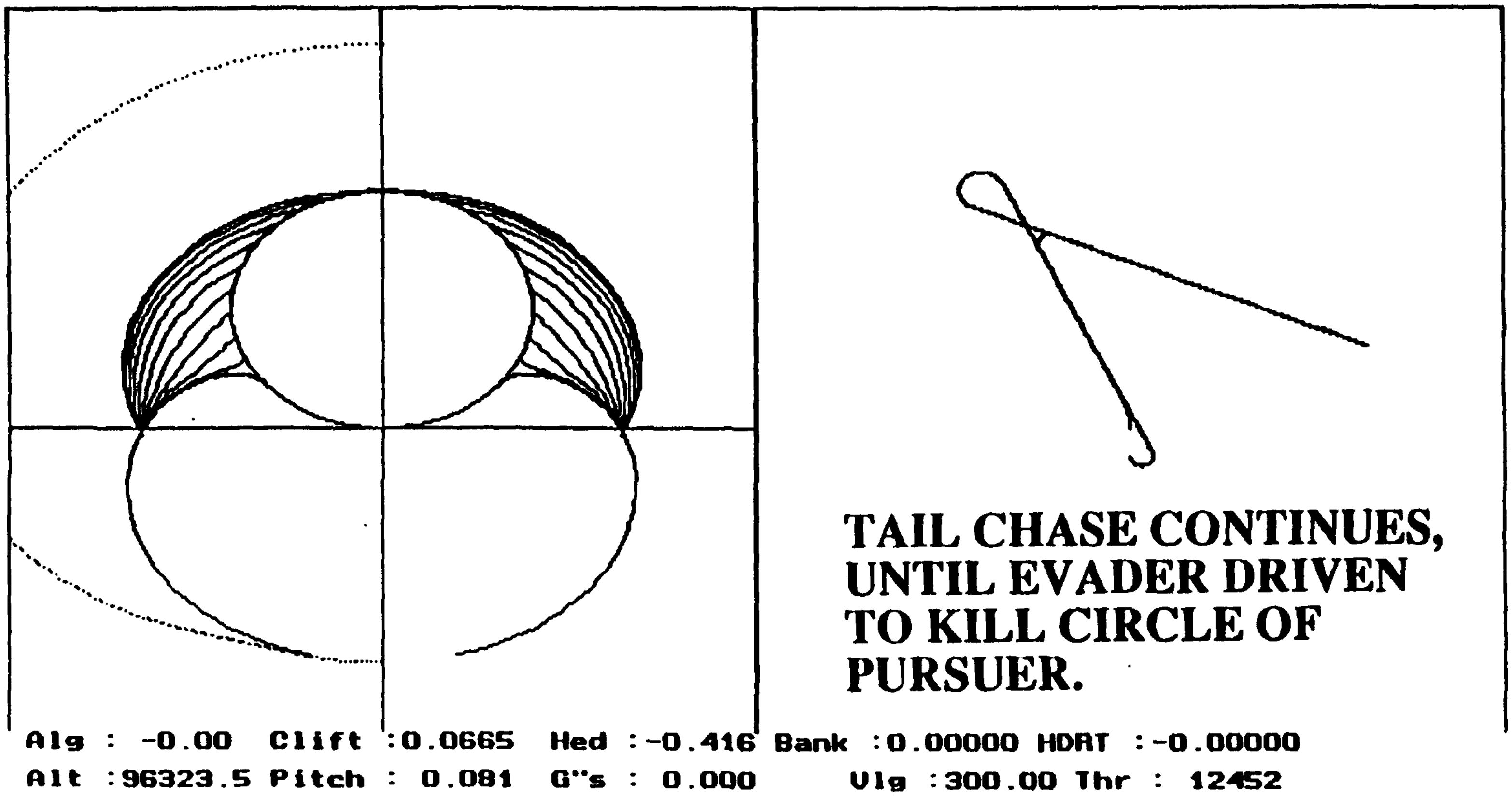

EVADER HAS REACHED
BOTTOM OF BARRIER.
BOTH BEGIN BREAKAWAY
MANEUVERS
Alg : -0.00 Clift :0.6000 Hed :-0.164 Bank :1.45962 HDRT :-0.29281
Alt :98745.1 Pitch : 0.063 G"s : 8.940 Vlg :300.01 Thr : 54711

A4


MUTUAL BREAKAWAY
CONTINUES
Alg : -0.00 Clift :0.0665 Hed :-0.416 Bank :-0.00004HDRT :0.00000
Alt :98508.1 Pitch : 0.081 G"s : 0.000 Vlg :300.02 Thr : 12452

A 5

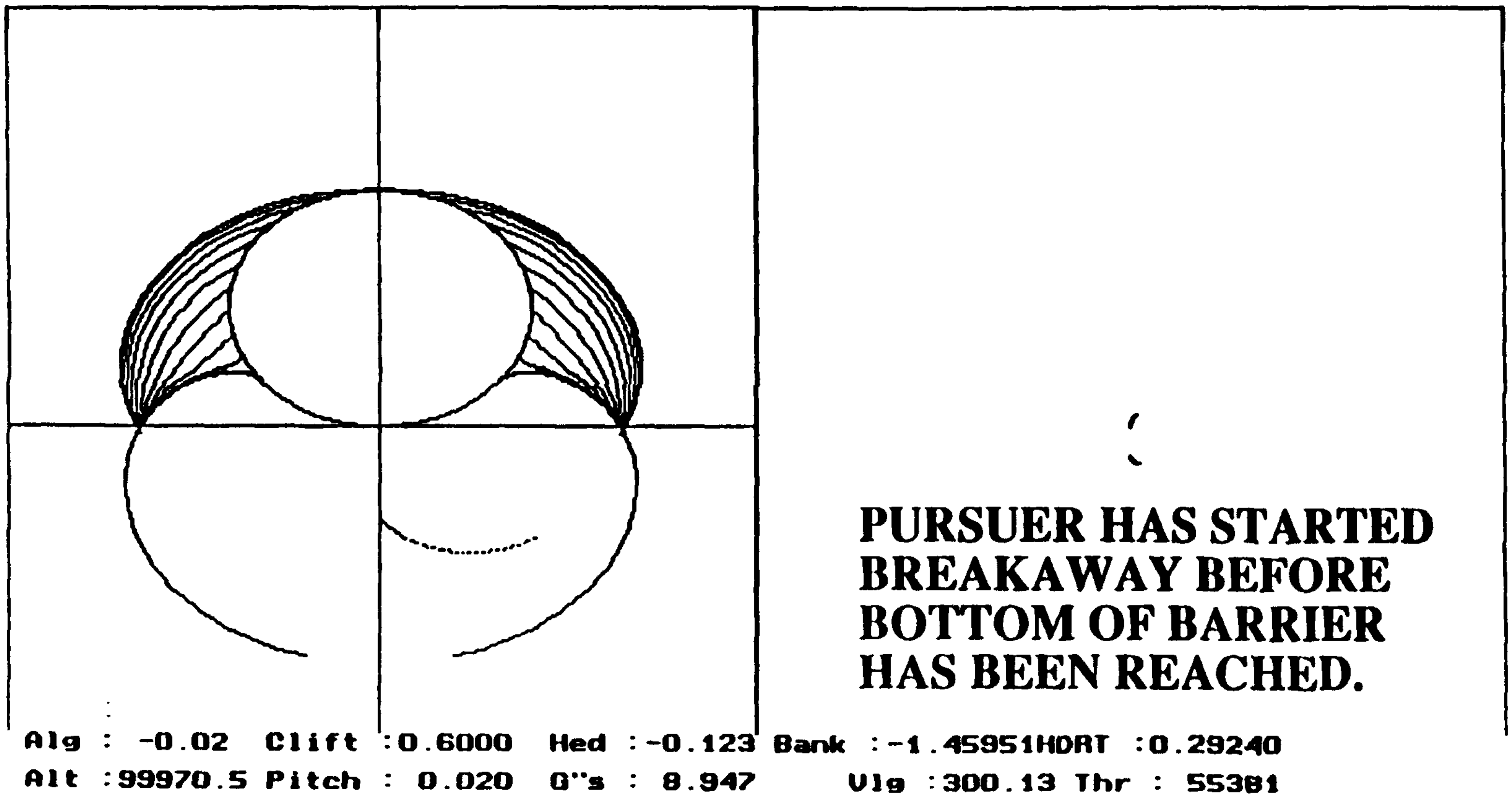

BREAKAWAY HAS ENDED, WITH PURSUER TAILCHASING THE EVADER
Alg : -0.00 Clift :0.0665 Hed :-0.416 Bank :-0.00000HDRT :0.00000
Alt :98022.6 Pitch : 0.081 G"s : 0.000 Vlg :300.01 Thr : 12452

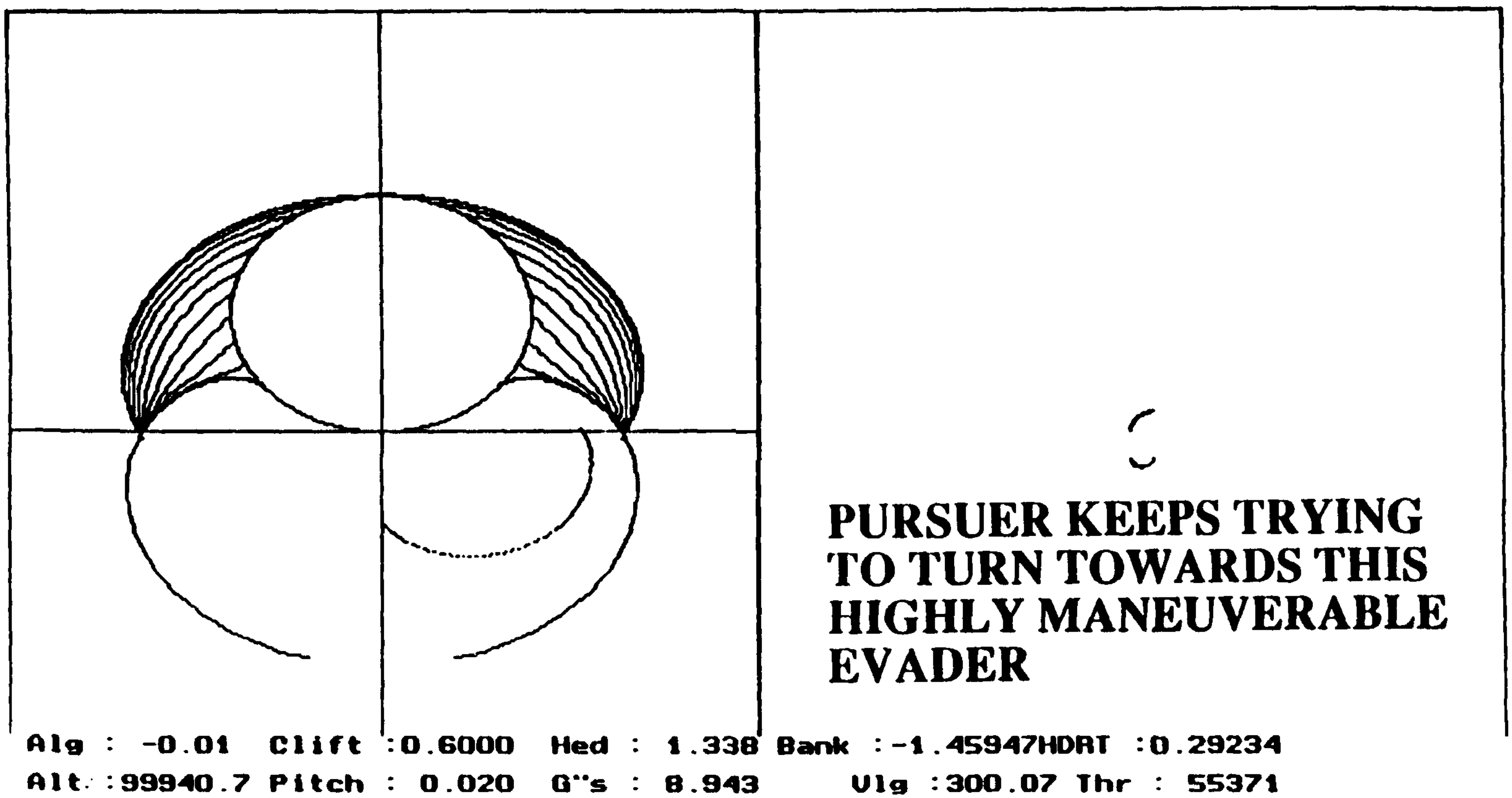


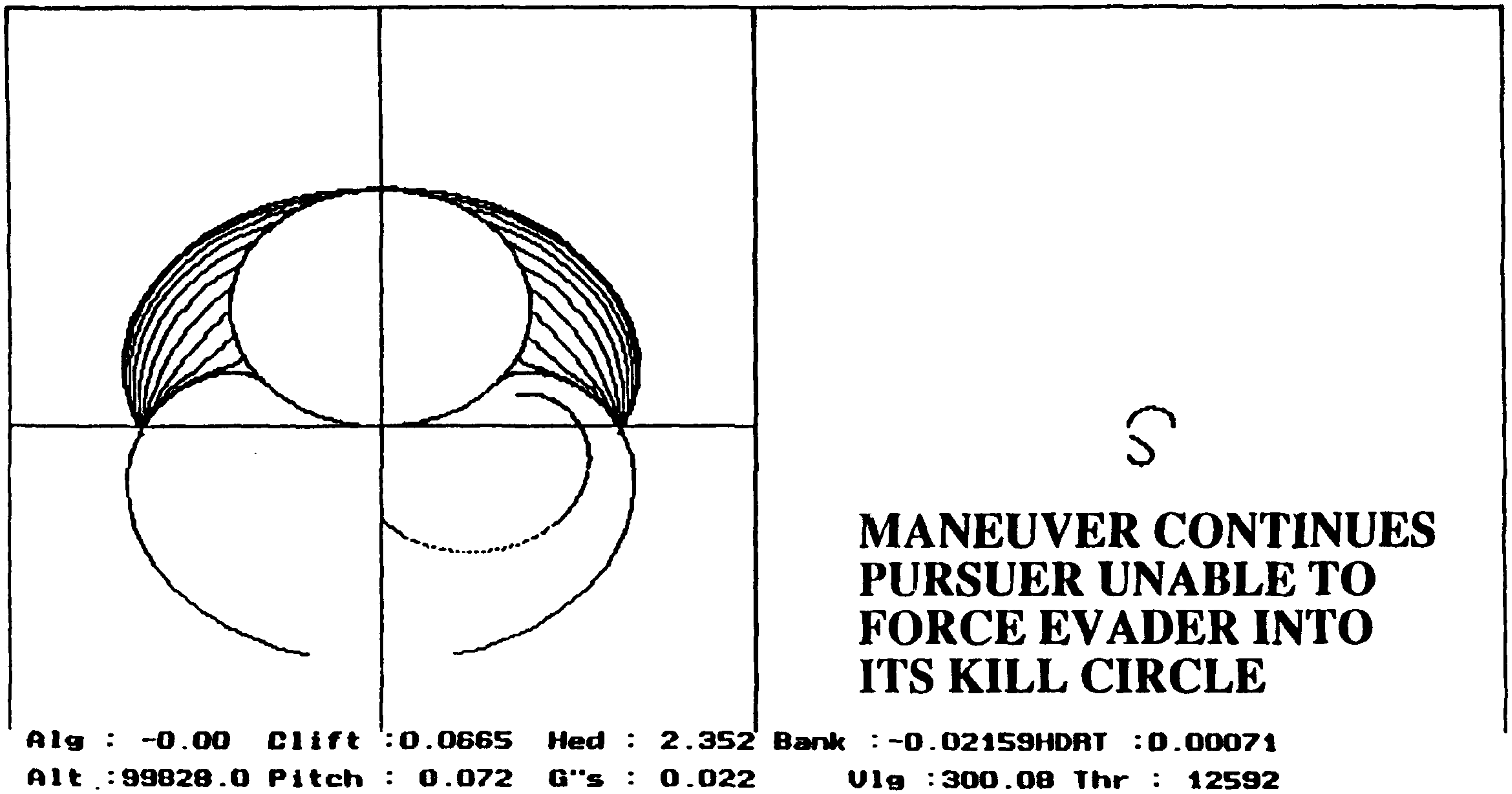
TAIL CHASE CONTINUES, UNTIL EVADER DRIVEN TO KILL CIRCLE OF PURSUER.

CASE OF PURSUER FLYING NON-OPTIMALLY, BY BREAKING AWAY FROM TAIL CHASE TOO SOON

A7




PURSUER KEEPS TRYING
TO TURN TOWARDS THIS
HIGHLY MANEUVERABLE
EVADER
Alg : -0.01 Clift :0.6000 Hed : 1.338 Bank :-1.45947HDRT :0.29234
Alt :99940.7 Pitch : 0.020 G"s : 8.943 Vlg :300.07 Thr : 55371


MANEUVER CONTINUES
PURSUER UNABLE TO
FORCE EVADER INTO
ITS KILL CIRCLE
Alg : -0.00 Clift :0.0665 Hed : 2.352 Bank :-0.02159HDRT :0.00071
Alt :99828.0 Pitch : 0.072 G"s : 0.022 Vlg :300.08 Thr : 12592